PROOF. It's enough to prove $(X_0 \times X_0) \sqcup \ldots \sqcup (X_n \times X_n) \in \operatorname{up}(x \times x)$ for every ultrafilter $x$, what follows from the fact that $x \sqsubseteq X_i$ for some $i$ and thus $x \times x \sqsubseteq X_i \times X_i$. □

PROPOSITION 1425. For finite tuples $X$, $Y$ of typed sets

$$(X_0 \times Y_0) \sqcup \ldots \sqcup (X_n \times Y_n) \sqsupseteq 1 \Leftrightarrow (X_0 \sqcap Y_0) \sqcup \ldots \sqcup (X_n \sqcap Y_n) = \top.$$

PROOF. $(X_0 \times Y_0) \sqcup \ldots \sqcup (X_n \times Y_n) \sqsupseteq 1 \Leftrightarrow ((X_0 \times Y_0) \sqcup \ldots \sqcup (X_n \times Y_n)) \sqcap 1 = 1 \Leftrightarrow ((X_0 \times Y_0) \sqcap 1) \sqcup \ldots \sqcup ((X_n \times Y_n) \sqcap 1) = 1 \Leftrightarrow \mathrm{id}_{X_0 \sqcap Y_0} \sqcup \ldots \sqcup \mathrm{id}_{X_n \sqcap Y_n} = 1 \Leftrightarrow \mathrm{id}_{(X_0 \sqcap Y_0) \sqcup \ldots \sqcup (X_n \sqcap Y_n)} = 1 \Leftrightarrow (X_0 \sqcap Y_0) \sqcup \ldots \sqcup (X_n \sqcap Y_n) = \top$. □

COROLLARY 1426.

$$\operatorname{up}^\Gamma 1 = \left\{ \frac{(X_0 \times Y_0) \sqcup \ldots \sqcup (X_n \times Y_n)}{n \in \mathbb{N}, \forall i \in n : X_i, Y_i \in \mathscr{T} U, (X_0 \sqcap Y_0) \sqcup \ldots \sqcup (X_n \sqcap Y_n) = \top} \right\}.$$

COROLLARY 1427. The predicate $(X_0 \sqcap Y_0) \sqcup \ldots \sqcup (X_n \sqcap Y_n) = \top$ for an element $(X_0 \times Y_0) \sqcup \ldots \sqcup (X_n \times Y_n)$ of $\Gamma$ does not depend on its representation $(X_0 \times Y_0) \sqcup \ldots \sqcup (X_n \times Y_n)$.

PROPOSITION 1428.

$$\operatorname{up}^\Gamma 1 = \bigcup \left\{ \frac{\operatorname{up}^\Gamma((X_0 \times X_0) \sqcup \ldots \sqcup (X_n \times X_n))}{n \in \mathbb{N}, \forall i \in n : X_i \in \mathscr{T} U, X_0 \sqcup \ldots \sqcup X_n = \top} \right\}.$$

PROOF. If $(X_0 \times Y_0) \sqcup \ldots \sqcup (X_n \times Y_n) \in \operatorname{up}^\Gamma 1$ then we have

$$(X_0 \times Y_0) \sqcup \ldots \sqcup (X_n \times Y_n) \sqsupseteq ((X_0 \sqcap Y_0) \times (X_0 \sqcap Y_0)) \sqcup \ldots \sqcup ((X_n \sqcap Y_n) \times (X_n \sqcap Y_n)) \in \operatorname{up}^\Gamma 1.$$

Thus

$$\operatorname{up}^\Gamma 1 \subseteq \bigcup \left\{ \frac{\operatorname{up}^\Gamma((X_0 \times X_0) \sqcup \ldots \sqcup (X_n \times X_n))}{n \in \mathbb{N}, \forall i \in n : X_i \in \mathscr{T} U, X_0 \sqcup \ldots \sqcup X_n = \top} \right\}.$$

The reverse inclusion is obvious. □

PROPOSITION 1429.

$$(\mathsf{RLD})_{\mathrm{in}} 1^{\mathsf{FCD}} = \prod^{\mathsf{RLD}} \left\{ \frac{(X_0 \times X_0) \sqcup \ldots \sqcup (X_n \times X_n)}{n \in \mathbb{N}, \forall i \in n : X_i \in \mathscr{T} U, X_0 \sqcup \ldots \sqcup X_n = \top} \right\}.$$

PROOF. By the diagram we have $(\mathsf{RLD})_{\mathrm{in}} 1^{\mathsf{FCD}} = \prod^{\mathsf{RLD}} \operatorname{up}^\Gamma 1$. So it follows from the previous proposition. □

PROPOSITION 1430. $\operatorname{up}^\Gamma (\mathsf{RLD})_{\mathrm{in}} 1^{\mathsf{FCD}} = \operatorname{up}^\Gamma 1$.

PROOF. If $K \in \operatorname{up}^\Gamma 1$ then $K \in \operatorname{up}^\Gamma((X_0 \times X_0) \sqcup \ldots \sqcup (X_n \times X_n))$ and thus $K \in \operatorname{up}^\Gamma (\mathsf{RLD})_{\mathrm{in}} 1^{\mathsf{FCD}}$ (see proposition 1424). Thus $\operatorname{up}^\Gamma 1 \subseteq \operatorname{up}^\Gamma (\mathsf{RLD})_{\mathrm{in}} 1^{\mathsf{FCD}}$. But $\operatorname{up}^\Gamma (\mathsf{RLD})_{\mathrm{in}} 1^{\mathsf{FCD}} \subseteq \operatorname{up}^\Gamma 1$ is obvious. □