PROPOSITION 1420. For a set S of binary relations

$$\forall X_0, \ldots, X_n \in S : \operatorname{up}(X_0 \sqcap^{\mathsf{FCD}} \cdots \sqcap^{\mathsf{FCD}} X_n) \subseteq S$$

does not imply that S is a funcoid base.

PROOF. Suppose for the contrary that it does imply. Then, because S is an upper set (as follows from the condition, taking $n = 0$), it implies that $S = \operatorname{up} f$ for a funcoid f, what contradicts to the above example. □

CONJECTURE 1421. Let $\forall X, Y \in S : \operatorname{up}(X \sqcap^{\mathsf{FCD}} Y) \subseteq S$.
Then

$$\forall X_0, \ldots, X_n \in S : \operatorname{up}(X_0 \sqcap^{\mathsf{FCD}} \cdots \sqcap^{\mathsf{FCD}} X_n) \subseteq S.$$

EXERCISE 1422. $\operatorname{up}(f_0 \sqcap^{\mathsf{FCD}} \ldots \sqcap^{\mathsf{FCD}} f_n) \subseteq \left\{ \frac{F_0 \sqcap \ldots \sqcap F_n}{F_0 \in \operatorname{up} f_0 \wedge \ldots \wedge F_n \in \operatorname{up} f_n} \right\}$ for every funcoids f_0, ..., f_n ($n \in \mathbb{N}$).

16.9. Some (example) values

I will do some calculations of particular funcoids and reloids.

First note that $\sqcap^{\mathsf{FCD}}$ can be decomposed (see below for a short easy proof):

$$f \sqcap^{\mathsf{FCD}} g = (\mathsf{FCD})(((\mathsf{RLD})_{\mathrm{in}} f \sqcap (\mathsf{RLD})_{\mathrm{in}} g).$$

The above is a more understandable decomposition of the operation $\sqcap^{\mathsf{FCD}}$ which behaves in strange way, mapping meet of two binary relations into a funcoid which is not a binary relation ($1^{\mathsf{FCD}} \sqcap^{\mathsf{FCD}} (\top \setminus 1^{\mathsf{FCD}}) = 1^{\mathsf{FCD}}_{\Omega}$).

The last formula is easy to prove (and proved above in the book) but the result is counter-intuitive.

More generally:

$$\bigsqcap^{\mathsf{FCD}} S = (\mathsf{FCD}) \bigsqcap^{\mathsf{RLD}} \langle (\mathsf{RLD})_{\mathrm{in}} \rangle^* S.$$

The above formulas follow from the fact that (FCD) is an upper adjoint and that $(\mathsf{FCD})(\mathsf{RLD})_{\mathrm{in}} f = f$ for every funcoid f.

Let FCD denote funcoids on a set U.

Consider a special case of the above formulas:

$$1^{\mathsf{FCD}} \sqcap^{\mathsf{FCD}} (\top \setminus 1^{\mathsf{FCD}}) = (\mathsf{FCD})((\mathsf{RLD})_{\mathrm{in}} 1^{\mathsf{FCD}} \sqcap (\mathsf{RLD})_{\mathrm{in}} (\top \setminus 1^{\mathsf{FCD}})). \quad (17)$$

We want to calculate terms of the formula (17) and more generally do some (probably useless) calculations for particular funcoids and reloids related to the above formula.

The left side is already calculated. The term $(\mathsf{RLD})_{\mathrm{in}} 1^{\mathsf{FCD}}$ which I call "thick equality" above is well understood. Let's compute $(\mathsf{RLD})_{\mathrm{in}} (\top \setminus 1^{\mathsf{FCD}})$.

PROPOSITION 1423. $(\mathsf{RLD})_{\mathrm{in}} (\top \setminus 1^{\mathsf{FCD}}) = \top \setminus 1^{\mathsf{FCD}}$.

PROOF. Consider funcoids on a set U. For any filters x and y (or without loss of generality ultrafilters x and y) we have:

$x \times^{\mathsf{FCD}} y \sqsubseteq \top \setminus 1^{\mathsf{FCD}} \Leftrightarrow$ (theorem 574 and the fact that funcoids are filters) $\Leftrightarrow$ $x \times^{\mathsf{FCD}} y \asymp 1^{\mathsf{FCD}} \Leftrightarrow \neg\big(x \, [1^{\mathsf{FCD}}] \, y\big) \Leftrightarrow x \asymp y \Rightarrow \exists X \in \operatorname{up} x, Y \in \operatorname{up} y : X \asymp Y$.

Thus $(\mathsf{RLD})_{\mathrm{in}} (\top \setminus 1^{\mathsf{FCD}}) = \bigsqcup \left\{ \frac{X \times Y}{X, Y \in \mathscr{T} U, X \asymp Y} \right\} = \top \setminus 1^{\mathsf{FCD}}$. □

So, we have:

$$1^{\mathsf{FCD}}_{\Omega} = 1^{\mathsf{FCD}} \sqcap^{\mathsf{FCD}} (\top \setminus 1^{\mathsf{FCD}}) = (\mathsf{RLD})_{\mathrm{in}} 1^{\mathsf{FCD}} \sqcap^{\mathsf{FCD}} (\top \setminus 1^{\mathsf{FCD}}).$$

PROPOSITION 1424. If $X_0 \sqcup \ldots \sqcup X_n = \top$ then $(X_0 \times X_0) \sqcup \ldots \sqcup (X_n \times X_n) \in \operatorname{up} (\mathsf{RLD})_{\mathrm{in}} 1^{\mathsf{FCD}}$.