Now it's obvious that $f \cap g = f \sqcap^{\mathsf{FCD}} g$. □

THEOREM 1410. The set of funcoids (from a given set A to a given set B) is with separable core.

PROOF. Let $f, g \in \mathsf{FCD}(A, B)$ (for some sets A,B).

Because filters on distributive lattices are with separable core, there exist $F, G \in \Gamma(A, B)$ such that $F \cap G = \emptyset$. Then by the previous theorem $F \sqcap^{\mathsf{FCD}} G = \bot$. □

THEOREM 1411. The coatoms of funcoids from a set A to a set B are exactly $(A \times B) \setminus (\{x\} \times \{y\})$ for $x \in A$, $y \in B$.

PROOF. That coatoms of $\Gamma(A, B)$ are exactly $(A \times B) \setminus (\{x\} \times \{y\})$ for $x \in A$, $y \in B$, is obvious. To show that coatoms of funcoids are the same, it remains to apply proposition 557. □

THEOREM 1412. The set of funcoids (for given A and B) is coatomic.

PROOF. Proposition 559. □

EXERCISE 1413. Prove that in general funcoids are not coatomistic.

16.8. Funcoid bases

This section will present mainly a counter-example against a statement you have not thought about anyway.

LEMMA 1414. If S is an upper set of principal funcoids, then $\bigsqcap^{\mathsf{FCD}}(S \cap \Gamma) = \bigsqcap^{\mathsf{FCD}} S$.

PROOF. $\bigsqcap^{\mathsf{FCD}}(S \cap \Gamma) \sqsupseteq \bigsqcap^{\mathsf{FCD}} S$ is obvious.

$\bigsqcap^{\mathsf{FCD}} S = \bigsqcap^{\mathsf{FCD}} \bigsqcap^{\mathsf{FCD}}_{K \in S} T_K \sqsupseteq \bigsqcap^{\mathsf{FCD}}(S \cap \Gamma)$. where $T_K \in \mathscr{P}(S \cap \Gamma)$. So $\bigsqcap^{\mathsf{FCD}}(S \cap \Gamma) = \bigsqcap^{\mathsf{FCD}} S$. □

THEOREM 1415. If S is a filter base on the set of binary relations then S is a base of $\bigsqcap^{\mathsf{FCD}} S$.

First prove a special case of our theorem to get the idea:

EXAMPLE 1416. Take the filter base $S = \left\{ \frac{\left\{ \frac{(x,y)}{|x-y|<\varepsilon} \right\}}{\varepsilon > 0} \right\}$ and $K = \left\{ \frac{(x,y)}{|x-y| < \exp x} \right\}$ where x and y range real numbers. Then $K \notin \operatorname{up} \bigsqcap^{\mathsf{FCD}} S$.

PROOF. Take a nontrivial ultrafilter x on $\mathbb{R}$. We can for simplicity assume $x \sqsubseteq \mathbb{Z}$.

$$\left\langle \bigsqcap^{\mathsf{FCD}} S \right\rangle x = \bigsqcap^{\mathscr{F}}_{L \in S} \langle L \rangle x = \bigsqcap^{\mathscr{F}}_{L \in S, X \in \operatorname{up} x} \langle L \rangle^* X = \bigsqcap^{\mathscr{F}}_{\varepsilon > 0, X \in \operatorname{up} x} \bigsqcup_{\alpha \in X}]\alpha - \varepsilon; \alpha + \varepsilon[.$$

$\langle K \rangle x = \bigsqcap^{\mathscr{F}}_{X \in \operatorname{up} x} \langle K \rangle^* X = \bigsqcap^{\mathscr{F}}_{X \in \operatorname{up} x} \bigsqcup_{\alpha \in X}]\alpha - \exp \alpha; \alpha + \exp \alpha[$.

Suppose for the contrary that $\langle K \rangle x \sqsupseteq \left\langle \bigsqcap^{\mathsf{FCD}} S \right\rangle x$.

Then

$\bigsqcup_{\alpha \in X}]\alpha - \exp \alpha; \alpha + \exp \alpha[\sqsupseteq \bigsqcap^{\mathscr{F}}_{\varepsilon > 0, X \in \operatorname{up} x} \bigsqcup_{\alpha \in X}]\alpha - \varepsilon; \alpha + \varepsilon[$ for every $X \in \operatorname{up} x$; thus by properties of generalized filter bases ($\left\{ \frac{\bigsqcup_{\alpha \in X}]\alpha - \varepsilon; \alpha + \varepsilon[}{\varepsilon > 0} \right\}$ is a filter base and even a chain)