

2°.

$$\begin{aligned} \left(\prod^{\text{RLD}} \text{up } g \right) \circ \left(\prod^{\text{RLD}} \text{up } f \right) &= \prod^{\text{RLD}} \left\{ \frac{G \circ F}{F \in \prod^{\text{RLD}} f, G \in \prod^{\text{RLD}} g} \right\} = \\ &= \prod^{\text{RLD}} \left\{ \frac{G \circ F}{F \in f, G \in g} \right\} = \prod^{\text{RLD}} \mathfrak{F}^{\Gamma(\text{Src } f, \text{Dst } g)} \left\{ \frac{G \circ F}{F \in f, G \in g} \right\} = \prod^{\text{RLD}} (g \circ f). \end{aligned}$$

So \prod^{RLD} preserves composition. That $\mathcal{A} \mapsto \Gamma(A, B) \cap \text{up } \mathcal{A}$ preserves composition follows from properties of bijections. \square

LEMMA 1391. Let A, B, C be sets.

- 1°. $\left(\prod^{\text{FCD}} \text{up } g \right) \circ \left(\prod^{\text{FCD}} \text{up } f \right) = \prod^{\text{FCD}} \text{up}(g \circ f)$ for every $f \in \mathfrak{F}\Gamma(A, B)$, $g \in \mathfrak{F}\Gamma(B, C)$;
- 2°. $\left(\text{up}^{\Gamma(B, C)} g \right) \circ \left(\text{up}^{\Gamma(A, B)} f \right) = \text{up}^{\Gamma(A, B)}(g \circ f)$ for every functors $f \in \text{FCD}(A, B)$ and $g \in \text{FCD}(B, C)$.

PROOF. It's enough to prove only the first formula, because of the bijection from lemma 1384.

Really:

$$\begin{aligned} \prod^{\text{FCD}} \text{up}(g \circ f) &= \prod^{\text{FCD}} \text{up} \prod^{\text{RLD}} \text{up}(g \circ f) = \\ &= \prod^{\text{FCD}} \text{up} \left(\prod^{\text{RLD}} \text{up } g \circ \prod^{\text{RLD}} \text{up } f \right) = (\text{FCD}) \left(\prod^{\text{RLD}} \text{up } g \circ \prod^{\text{RLD}} \text{up } f \right) = \\ &= \left((\text{FCD}) \prod^{\text{RLD}} \text{up } g \right) \circ \left((\text{FCD}) \prod^{\text{RLD}} \text{up } f \right) = \\ &= \left(\prod^{\text{FCD}} \text{up} \prod^{\text{RLD}} \text{up } g \right) \circ \left(\prod^{\text{FCD}} \text{up} \prod^{\text{RLD}} \text{up } f \right) = \\ &= \left(\prod^{\text{FCD}} \text{up } g \right) \circ \left(\prod^{\text{FCD}} \text{up } f \right). \end{aligned}$$

\square

COROLLARY 1392. $(h \circ g) \circ f = h \circ (g \circ f)$ for every $f \in \mathfrak{F}(\Gamma(A, B))$, $g \in \mathfrak{F}(\Gamma(B, C))$, $h \in \mathfrak{F}(\Gamma(C, D))$ for every sets A, B, C, D .

LEMMA 1393. $\Gamma(A, B) \cap \text{GR } f$ is a filter on the lattice $\Gamma(A, B)$ for every reloid $f \in \text{RLD}(A, B)$.

PROOF. That it is an upper set, is obvious. If $A, B \in \Gamma(A, B) \cap \text{GR } f$ then $A, B \in \Gamma(A, B)$ and $A, B \in \text{GR } f$. Thus $A \cap B \in \Gamma(A, B) \cap \text{GR } f$. \square

PROPOSITION 1394. If $Y \in \text{up}\langle f \rangle \mathcal{X}$ for a functor f then there exists $A \in \text{up } \mathcal{X}$ such that $Y \in \text{up}\langle f \rangle A$.

PROOF. $Y \in \text{up} \prod_{A \in \text{up } a}^{\mathcal{F}} \langle f \rangle A$. So by properties of generalized filter bases, there exists $A \in \text{up } a$ such that $Y \in \text{up}\langle f \rangle A$. \square

LEMMA 1395. $(\text{FCD})f = \prod^{\text{FCD}}(\Gamma(A, B) \cap \text{GR } f)$ for every reloid $f \in \text{RLD}(A, B)$.