3°⊆4°.

$$\bigcup_{X\in S}(X\times Y_X)=\bigcup_{X\in S}\left(X\times\bigcup\left(\mathfrak{R}\left(\left\{\frac{Y_X}{X\in S}\right\}\right)\cap\mathscr{P}Y_X\right)\right)=$$

$$\bigcup_{X\in S}\left(X\times\bigcup\left\{\frac{Y'\in\mathfrak{R}(\{\frac{Y_X}{X\in S}\})}{Y'\subseteq Y_X}\right\}\right)=$$

$$\bigcup_{X\in S}\left(X\times\bigcup\left\{\frac{Y'\in\mathfrak{R}(\{\frac{Y_X}{X\in S}\})}{(X,Y')\in\sigma}\right\}\right)=\bigcup_{(X,Y)\in\sigma}(X\times Y)$$

where σ is a relation between S and $\mathfrak{R}(\{\frac{Y_X}{X\in S}\})$, and $(X,Y')\in\sigma\Leftrightarrow Y'\subseteq Y_X$.

5°⊆1°. Obvious.

3°⊆5°. Let $Q=\bigcup_{X\in S}(X\times Y_X)=\bigcup_{i=0,\dots,n-1}(X_i\times Y_i)$ for a partition $S=\{X_0,\dots,X_{n-1}\}$ of A. Then $Q=\bigcap_{i=0,\dots,n-1}(X_i\times Y_i\cup\overline{X_i}\times B)$.

□

EXERCISE 1379. Formulate the duals of these sets.

PROPOSITION 1380. $\Gamma(A,B)$ is a boolean lattice, a sublattice of the lattice $\mathscr{P}(A\times B)$.

PROOF. That it's a sublattice is obvious. That it has complement, is also obvious. Distributivity follows from distributivity of $\mathscr{P}(A\times B)$. □

16.3. Before the diagram

Next we will prove the below theorem 1396 (the theorem with a diagram). First we will present parts of this theorem as several lemmas, and then then state a statement about the diagram which concisely summarizes the lemmas (and their easy consequences).

Below for simplicity we will equate reloids with their graphs (that is with filters on binary cartesian products).

OBVIOUS 1381. $\operatorname{up}^{\Gamma(\operatorname{Src}f,\operatorname{Dst}f)}f=(\operatorname{up}f)\cap\Gamma$ for every reloid f.

CONJECTURE 1382. $\upuparrows^{\mathfrak{F}(\mathfrak{B})}\operatorname{up}^{\mathfrak{A}}\mathcal{X}$ is not a filter for some filter $\mathcal{X}\in\mathfrak{F}\Gamma(A,B)$ for some sets A, B.

REMARK 1383. About this conjecture see also:

- http://goo.gl/DHyuuU
- http://goo.gl/4a6wY6

LEMMA 1384. Let A, B be sets. The following are mutually inverse order isomorphisms between $\mathfrak{F}\Gamma(A,B)$ and $\mathsf{FCD}(A,B)$:

1°. $\mathcal{A}\mapsto\prod^{\mathsf{FCD}}\operatorname{up}\mathcal{A}$;
2°. $f\mapsto\operatorname{up}^{\Gamma(A,B)}f$.

PROOF. Let's prove that $\operatorname{up}^{\Gamma(A,B)}f$ is a filter for every funcoid f. We need to prove that $P\cap Q\in\operatorname{up}f$ whenever

$$P=\bigcap_{i=0,\dots,n-1}(X_i\times Y_i\cup\overline{X_i}\times B)\quad\text{and}\quad Q=\bigcap_{j=0,\dots,m-1}\left(X'_j\times Y'_j\cup\overline{X'_j}\times B\right).$$

This follows from $P\in\operatorname{up}f\Leftrightarrow\forall i\in 0,\dots,n-1:\langle f\rangle X_i\subseteq Y_i$ and likewise for Q, so having $\langle f\rangle(X_i\cap X'_j)\subseteq Y_i\cap Y'_j$ for every $i=0,\dots,n-1$ and $j=0,\dots,m-1$. From this it follows

$$((X_i\cap X'_j)\times(Y_i\cap Y'_j))\cup\left(\overline{X_i\cap X'_j}\times B\right)\supseteq f$$