CONJECTURE 1362. $\mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B} \sqsubset \mathcal{A} \ltimes \mathcal{B}$ for some filters $\mathcal{A}$, $\mathcal{B}$.

A stronger conjecture:

CONJECTURE 1363. $\mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B} \sqsubset \mathcal{A} \ltimes \mathcal{B} \sqsubset \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$ for some filters $\mathcal{A}$, $\mathcal{B}$. Particularly, is this formula true for $\mathcal{A} = \mathcal{B} = \Delta \sqcap \uparrow^{\mathbb{R}}]0; +\infty[$?

The above conjecture is similar to Fermat Last Theorem as having no value by itself but being somehow challenging to prove it (not expected to be as hard as FLT however).

EXAMPLE 1364. $\mathcal{A} \ltimes \mathcal{B} \sqsubset \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$ for some filters $\mathcal{A}$, $\mathcal{B}$.

PROOF. It's enough to prove $\mathcal{A} \ltimes \mathcal{B} \neq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$.
Let $\Delta_+ = \Delta \sqcap \uparrow^{\mathbb{R}}]0; +\infty[$. Let $\mathcal{A} = \mathcal{B} = \Delta_+$.
Let $K = (\leq)|_{\mathbb{R}\times\mathbb{R}}$.
Obviously $K \notin \operatorname{up}(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B})$.
$\mathcal{A} \ltimes \mathcal{B} \sqsubseteq \uparrow^{\mathsf{RLD}(\operatorname{Base}(\mathcal{A}),\operatorname{Base}(\mathcal{B}))} K$ and thus $K \in \operatorname{up}(\mathcal{A} \ltimes \mathcal{B})$ because

$$\uparrow^{\mathsf{FCD}(\operatorname{Base}(\mathcal{A}),\operatorname{Base}(\mathcal{B}))} K \sqsupseteq \Delta_+ \times^{\mathsf{FCD}} \uparrow B = \mathcal{A} \times^{\mathsf{FCD}} \uparrow B$$

for $B =]0; +\infty[$ because for every $X \in \partial\Delta_+$ there is $x \in X$ such that $x \in]0; \epsilon[$ (for every positive ϵ) and thus $]\epsilon; +\infty[\subseteq \langle K\rangle^* \{x\}$ so having

$$\langle K\rangle^* X =]0; +\infty[\in \operatorname{GR}\left\langle \Delta_+ \times^{\mathsf{FCD}} \uparrow B\right\rangle^* X.$$

Thus $\mathcal{A} \ltimes \mathcal{B} \neq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$. □

EXAMPLE 1365. $\mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B} \sqsubset \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$ for some filters $\mathcal{A}$, $\mathcal{B}$.

PROOF. This follows from the above example. □

CONJECTURE 1366. $(\mathcal{A} \ltimes \mathcal{B}) \sqcap (\mathcal{A} \rtimes \mathcal{B}) \neq \mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B}$ for some filters $\mathcal{A}$, $\mathcal{B}$.

(Earlier I presented a proof of the negation of this conjecture, but it was in error.)

EXAMPLE 1367. $(\mathcal{A} \ltimes \mathcal{B}) \sqcup (\mathcal{A} \rtimes \mathcal{B}) \sqsubset \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$ for some filters $\mathcal{A}$, $\mathcal{B}$.

PROOF. (based on [8]) Let $\mathcal{A} = \mathcal{B} = \Omega(\mathbb{N})$. It's enough to prove $(\mathcal{A} \ltimes \mathcal{B}) \sqcup (\mathcal{A} \rtimes \mathcal{B}) \neq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$.
Let $X \in \operatorname{up} \mathcal{A}$, $Y \in \operatorname{up} \mathcal{B}$ that is $X \in \Omega(\mathbb{N})$, $Y \in \Omega(\mathbb{N})$.
Removing one element x from X produces a set P. Removing one element y from Y produces a set Q. Obviously $P \in \Omega(\mathbb{N})$, $Q \in \Omega(\mathbb{N})$.
Obviously $(P \times \mathbb{N}) \cup (\mathbb{N} \times Q) \in \operatorname{up}((\mathcal{A} \ltimes \mathcal{B}) \sqcup (\mathcal{A} \rtimes \mathcal{B}))$.
$(P \times \mathbb{N}) \cup (\mathbb{N} \times Q) \not\supseteq X \times Y$ because $(x, y) \in X \times Y$ but $(x, y) \notin (P \times \mathbb{N}) \cup (\mathbb{N} \times Q)$ for every $X \in \operatorname{up} \mathcal{A}$, $Y \in \operatorname{up} \mathcal{B}$.
Thus some $(P \times \mathbb{N}) \cup (\mathbb{N} \times Q) \notin \operatorname{up}(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B})$ by properties of filter bases. □

EXAMPLE 1368. $(\mathsf{RLD})_{\mathrm{out}}(\mathsf{FCD})f \neq f$ for some convex reloid f.

PROOF. Let $f = \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$ where $\mathcal{A}$ and B are from example 1365.
$(\mathsf{FCD})(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) = \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}$ by proposition 1071.
So $(\mathsf{RLD})_{\mathrm{out}}(\mathsf{FCD})(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) = (\mathsf{RLD})_{\mathrm{out}}(\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) = \mathcal{A} \times^{\mathsf{RLD}}_F \mathcal{B} \neq \mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}$. □