PROOF. Let $f = 1^{\mathsf{FCD}}_{\mathbb{N}}$. Then $(\mathsf{RLD})_{\mathrm{in}} f = \bigsqcup_{a \in \operatorname{atoms}^{\mathscr{F}(\mathbb{N})}} (a \times^{\mathsf{RLD}} a)$ and $(\mathsf{RLD})_{\mathrm{out}} f = 1^{\mathsf{RLD}}_{\mathbb{N}}$. But we have shown above $a \times^{\mathsf{RLD}} a \not\sqsubseteq 1^{\mathsf{RLD}}_{\mathbb{N}}$ for non-trivial ultrafilter a, and so $(\mathsf{RLD})_{\mathrm{in}} f \not\sqsubseteq (\mathsf{RLD})_{\mathrm{out}} f$. □

PROPOSITION 1345. $1^{\mathsf{FCD}}_{\mathfrak{U}} \sqcap \uparrow^{\mathsf{FCD}(\mathfrak{U},\mathfrak{U})} ((\mathfrak{U} \times \mathfrak{U}) \setminus \mathrm{id}_{\mathfrak{U}}) = \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \neq \bot^{\mathsf{FCD}(\mathfrak{U},\mathfrak{U})}$ for every infinite set $\mathfrak{U}$.

PROOF. Note that $\left\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \right\rangle \mathcal{X} = \mathcal{X} \sqcap \Omega(\mathfrak{U})$ for every filter $\mathcal{X}$ on $\mathfrak{U}$.

Let $f = 1^{\mathsf{FCD}}_{\mathfrak{U}}$, $g = \uparrow^{\mathsf{FCD}(\mathfrak{U},\mathfrak{U})} ((\mathfrak{U} \times \mathfrak{U}) \setminus \mathrm{id}_{\mathfrak{U}})$.

Let x be a non-trivial ultrafilter on $\mathfrak{U}$. If $X \in \operatorname{up} x$ then $\operatorname{card} X \geq 2$ (In fact, X is infinite but we don't need this.) and consequently $\langle g \rangle^* X = \top^{\mathscr{F}(\mathfrak{U})}$. Thus $\langle g \rangle x = \top^{\mathscr{F}(\mathfrak{U})}$. Consequently

$$\langle f \sqcap g \rangle x = \langle f \rangle x \sqcap \langle g \rangle x = x \sqcap \top^{\mathscr{F}(\mathfrak{U})} = x.$$

Also $\left\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \right\rangle x = x \sqcap \Omega(\mathfrak{U}) = x$.

Let now x be a trivial ultrafilter. Then $\langle f \rangle x = x$ and $\langle g \rangle x = \top^{\mathscr{F}(\mathfrak{U})} \setminus x$. So

$$\langle f \sqcap g \rangle x = \langle f \rangle x \sqcap \langle g \rangle x = x \sqcap (\top^{\mathscr{F}(\mathfrak{U})} \setminus x) = \bot^{\mathscr{F}(\mathfrak{U})}.$$

Also $\left\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \right\rangle x = x \sqcap \Omega(\mathfrak{U}) = \bot^{\mathscr{F}(\mathfrak{U})}$.

So $\langle f \sqcap g \rangle x = \left\langle \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})} \right\rangle x$ for every ultrafilter x on $\mathfrak{U}$. Thus $f \sqcap g = \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathfrak{U})}$. □

EXAMPLE 1346. There exist binary relations f and g such that $\uparrow^{\mathsf{FCD}(A,B)} f \sqcap \uparrow^{\mathsf{FCD}(A,B)} g \neq \uparrow^{\mathsf{FCD}(A,B)} (f \cap g)$ for some sets A, B such that $f, g \subseteq A \times B$.

PROOF. From the proposition above. □

EXAMPLE 1347. There exists a principal funcoid which is not a complemented element of the lattice of funcoids.

PROOF. I will prove that quasi-complement of the funcoid $1^{\mathsf{FCD}}_{\mathbb{N}}$ is not its complement (it is enough by proposition 145). We have:

$$\begin{aligned}
(1^{\mathsf{FCD}}_{\mathbb{N}})^* &= \\
\bigsqcup \left\{ \frac{c \in \mathsf{FCD}(\mathbb{N},\mathbb{N})}{c \asymp 1^{\mathsf{FCD}}_{\mathbb{N}}} \right\} &\sqsupseteq \\
\bigsqcup \left\{ \frac{\uparrow^{\mathbb{N}} \{\alpha\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\beta\}}{\alpha, \beta \in \mathbb{N}, \uparrow^{\mathbb{N}} \{\alpha\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\beta\} \asymp 1^{\mathsf{FCD}}_{\mathbb{N}}} \right\} &= \\
\bigsqcup \left\{ \frac{\uparrow^{\mathbb{N}} \{\alpha\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{N}} \{\beta\}}{\alpha, \beta \in \mathbb{N}, \alpha \neq \beta} \right\} &= \\
\uparrow^{\mathsf{FCD}(\mathbb{N},\mathbb{N})} \bigcup \left\{ \frac{\{\alpha\} \times \{\beta\}}{\alpha, \beta \in \mathbb{N}, \alpha \neq \beta} \right\} &= \\
\uparrow^{\mathsf{FCD}(\mathbb{N},\mathbb{N})} (\mathbb{N} \times \mathbb{N} \setminus \mathrm{id}_{\mathbb{N}}) &
\end{aligned}$$

(used corollary 920). But by proved above $(1^{\mathsf{FCD}}_{\mathbb{N}})^* \sqcap 1^{\mathsf{FCD}}_{\mathbb{N}} \neq \bot^{\mathscr{F}(\mathbb{N})}$. □

EXAMPLE 1348. There exists a funcoid h such that $\operatorname{up} h$ is not a filter.

PROOF. Consider the funcoid $h = \mathrm{id}^{\mathsf{FCD}}_{\Omega(\mathbb{N})}$. We have (from the proof of proposition 1345) that $f \in \operatorname{up} h$ and $g \in \operatorname{up} h$, but $f \sqcap g \notin \operatorname{up} h$. □

EXAMPLE 1349. There exists a funcoid $h \neq \bot^{\mathsf{FCD}(A,B)}$ such that $(\mathsf{RLD})_{\mathrm{out}} h = \bot^{\mathsf{RLD}(A,B)}$.