Proof. Take $\mathbb{R}_+ = \left\{ \frac{x\in\mathbb{R}}{x>0} \right\}$, $\mathcal{A} = \Delta$, $T = \left\{ \frac{\uparrow\{x\}}{x\in\mathbb{R}_+} \right\}$ where $\uparrow = \uparrow^{\mathbb{R}}$.

$\bigsqcup T = \uparrow \mathbb{R}_+$; $\mathcal{A} \times^{\mathsf{RLD}} \bigsqcup T = \Delta \times^{\mathsf{RLD}} \uparrow \mathbb{R}_+$.

$\bigsqcup_{\mathcal{B}\in T}(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) = \bigsqcup_{x\in\mathbb{R}_+}(\Delta \times^{\mathsf{RLD}} \uparrow \{x\})$.

We'll prove that $\bigsqcup_{x\in\mathbb{R}_+}(\Delta \times^{\mathsf{RLD}} \uparrow \{x\}) \neq \Delta \times^{\mathsf{RLD}} \uparrow \mathbb{R}_+$.

Consider $K = \bigcup_{x\in\mathbb{R}_+}(\{x\}\times]-1/x; 1/x[)$.

$K \in \mathrm{up}(\Delta \times^{\mathsf{RLD}} \uparrow \{x\})$ and thus $K \in \mathrm{up} \bigsqcup_{x\in\mathbb{R}_+}(\Delta \times^{\mathsf{RLD}} \uparrow \{x\})$. But $K \notin \mathrm{up}(\Delta \times^{\mathsf{RLD}} \uparrow \mathbb{R}_+)$. □

Theorem 1338. For a filter a we have $a \times^{\mathsf{RLD}} a \sqsubseteq 1^{\mathsf{RLD}}_{\mathrm{Base}(a)}$ only in the case if $a = \bot^{\mathscr{F}(\mathrm{Base}(a))}$ or a is a trivial ultrafilter.

Proof. If $a \times^{\mathsf{RLD}} a \sqsubseteq 1^{\mathsf{RLD}}_{\mathrm{Base}(a)}$ then there exists $m \in \mathrm{up}(a \times^{\mathsf{RLD}} a)$ such that $m \sqsubseteq 1^{\mathbf{Rel}}_{\mathrm{Base}(a)}$. Consequently there exist $A, B \in \mathrm{up}\, a$ such that $A \times B \sqsubseteq 1^{\mathbf{Rel}}_{\mathrm{Base}(a)}$ what is possible only in the case when $\uparrow A = \uparrow B = a$ is trivial a ultrafilter or the least filter. □

Corollary 1339. Reloidal product of a non-trivial atomic filter with itself is non-atomic.

Proof. Obviously $(a \times^{\mathsf{RLD}} a) \sqcap 1^{\mathsf{RLD}}_{\mathrm{Base}(a)} \neq \bot^{\mathsf{RLD}}$ and $(a \times^{\mathsf{RLD}} a) \sqcap 1^{\mathsf{RLD}}_{\mathrm{Base}(a)} \sqsubset a \times^{\mathsf{RLD}} a$. □

Example 1340. There exist two atomic reloids whose composition is non-atomic and non-empty.

Proof. Let a be a non-trivial ultrafilter on $\mathbb{N}$ and $x \in \mathbb{N}$. Then

$$(a \times^{\mathsf{RLD}} \uparrow^{\mathbb{N}} \{x\}) \circ (\uparrow^{\mathbb{N}} \{x\} \times^{\mathsf{RLD}} a) = \prod^{\mathsf{RLD}(\mathbb{N},\mathbb{N})}_{A\in a} ((A \times \{x\}) \circ (\{x\} \times A) = \prod^{\mathsf{RLD}(\mathbb{N},\mathbb{N})}_{A\in a} (A \times A) = a \times^{\mathsf{RLD}} a$$

is non-atomic despite of $a \times^{\mathsf{RLD}} \uparrow^{\mathbb{N}} \{x\}$ and $\uparrow^{\mathbb{N}} \{x\} \times^{\mathsf{RLD}} a$ are atomic. □

Example 1341. There exists non-monovalued atomic reloid.

Proof. From the previous example it follows that the atomic reloid $\uparrow^{\mathbb{N}} \{x\} \times^{\mathsf{RLD}} a$ is not monovalued. □

Example 1342. Non-convex reloids exist.

Proof. Let a be a non-trivial ultrafilter. Then $\mathrm{id}^{\mathsf{RLD}}_a$ is non-convex. This follows from the fact that only reloidal products which are below $1^{\mathsf{RLD}}_{\mathrm{Base}(a)}$ are reloidal products of ultrafilters and $\mathrm{id}^{\mathsf{RLD}}_a$ is not their join. □

Example 1343. There exists (atomic) composable funcoids f and g such that

$$H \in \mathrm{up}(g \circ f) \nRightarrow \exists F \in \mathrm{up}\, f, G \in \mathrm{up}\, g : H \sqsupseteq G \circ F.$$

Proof. Let a be a nontrivial ultrafilter and p be an arbitrary point, $f = a \times^{\mathsf{FCD}} \{p\}$, $g = \{p\} \times^{\mathsf{FCD}} a$. Then $g \circ f = a \times^{\mathsf{FCD}} a$. Take $H = 1$. Let $F \in \mathrm{up}\, f$ and $G \in \mathrm{up}\, g$. We have $F \in \mathrm{up}(A_0 \times^{\mathsf{FCD}} \{p\})$, $G \in \mathrm{up}(\{p\} \times^{\mathsf{FCD}} A_1)$ where $A_0, A_1 \in \mathrm{up}\, a$ (take $A_0 = \langle F\rangle^{*} @\{p\}$ and similarly for A_1). Thus $G \circ F \sqsupseteq A_0 \times A_1$ and so $H \notin \mathrm{up}(G \circ F)$. □

Example 1344. $(\mathsf{RLD})_{\mathrm{in}} f \neq (\mathsf{RLD})_{\mathrm{out}} f$ for a funcoid f.