CHAPTER 15

Counter-examples about funcoids and reloids

For further examples we will use the filter defined by the formula

$$\Delta = \bigsqcap^{\mathscr{F}(\mathbb{R})} \left\{ \frac{]-\epsilon;\epsilon[}{\epsilon \in \mathbb{R}, \epsilon > 0} \right\}.$$

I will denote $\Omega(A)$ the Fréchet filter on a set A.

EXAMPLE 1333. There exist a funcoid f and a set S of funcoids such that $f \sqcap \bigsqcup S \neq \bigsqcup \langle f \sqcap \rangle^* S$.

PROOF. Let $f = \Delta \times^{\mathsf{FCD}} \uparrow^{\mathscr{F}(\mathbb{R})} \{0\}$ and $S = \left\{ \frac{\uparrow^{\mathsf{FCD}(\mathbb{R},\mathbb{R})} (]\epsilon;+\infty[\times\{0\})}{\epsilon\in\mathbb{R},\epsilon>0} \right\}$. Then

$$f \sqcap \bigsqcup S = (\Delta \times^{\mathsf{FCD}} \uparrow^{\mathscr{F}(\mathbb{R})} \{0\}) \sqcap \uparrow^{\mathsf{FCD}(\mathbb{R},\mathbb{R})} (]0;+\infty[\times\{0\}) = (\Delta \sqcap \uparrow^{\mathscr{F}(\mathbb{R})}]0;+\infty[) \times^{\mathsf{FCD}} \uparrow^{\mathscr{F}(\mathbb{R})} \{0\} \neq \bot^{\mathsf{FCD}(\mathbb{R},\mathbb{R})}$$

while $\bigsqcup \langle f \sqcap \rangle^* S = \bigsqcup \{ \bot^{\mathsf{FCD}(\mathbb{R},\mathbb{R})} \} = \bot^{\mathsf{FCD}(\mathbb{R},\mathbb{R})}$. □

EXAMPLE 1334. There exist a set R of funcoids and a funcoid f such that $f \circ \bigsqcup R \neq \bigsqcup \langle f \circ \rangle^* R$.

PROOF. Let $f = \Delta \times^{\mathsf{FCD}} \uparrow^{\mathscr{F}(\mathbb{R})} \{0\}$, $R = \left\{ \frac{\uparrow^{\mathbb{R}} \{0\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{R}}]\epsilon;+\infty[}{\epsilon\in\mathbb{R},\epsilon>0} \right\}$.

We have $\bigsqcup R = \uparrow^{\mathbb{R}} \{0\} \times^{\mathsf{FCD}} \uparrow^{\mathbb{R}}]0;+\infty[$; $f \circ \bigsqcup R = \uparrow^{\mathsf{FCD}(\mathbb{R},\mathbb{R})} (\{0\} \times \{0\}) \neq \bot^{\mathsf{FCD}(\mathbb{R},\mathbb{R})}$ and $\bigsqcup \langle f \circ \rangle^* R = \bigsqcup \{ \bot^{\mathsf{FCD}(\mathbb{R},\mathbb{R})} \} = \bot^{\mathsf{FCD}(\mathbb{R},\mathbb{R})}$. □

EXAMPLE 1335. There exist a set R of reloids and a reloid f such that $f \circ \bigsqcup R \neq \bigsqcup \langle f \circ \rangle^* R$.

PROOF. Let $f = \Delta \times^{\mathsf{RLD}} \uparrow^{\mathscr{F}(\mathbb{R})} \{0\}$, $R = \left\{ \frac{\uparrow^{\mathbb{R}} \{0\} \times^{\mathsf{RLD}} \uparrow^{\mathbb{R}}]\epsilon;+\infty[}{\epsilon\in\mathbb{R},\epsilon>0} \right\}$.

We have $\bigsqcup R = \uparrow^{\mathbb{R}} \{0\} \times^{\mathsf{RLD}} \uparrow^{\mathbb{R}}]0;+\infty[$; $f \circ \bigsqcup R = \uparrow^{\mathsf{RLD}(\mathbb{R},\mathbb{R})} (\{0\} \times \{0\}) \neq \bot^{\mathsf{RLD}(\mathbb{R},\mathbb{R})}$ and $\bigsqcup \langle f \circ \rangle^* R = \bigsqcup \{ \bot^{\mathsf{RLD}(\mathbb{R},\mathbb{R})} \} = \bot^{\mathsf{RLD}(\mathbb{R},\mathbb{R})}$. □

EXAMPLE 1336. There exist a set R of funcoids and filters $\mathcal{X}$ and $\mathcal{Y}$ such that

1°. $\mathcal{X} \left[\bigsqcup R \right] \mathcal{Y} \wedge \nexists f \in R : \mathcal{X} [f] \mathcal{Y}$;

2°. $\langle \bigsqcup R \rangle \mathcal{X} \sqsupset \bigsqcup \left\{ \frac{\langle f \rangle \mathcal{X}}{f \in R} \right\}$.

PROOF.

1°. Take $\mathcal{X} = \Delta$ and $\mathcal{Y} = \top^{\mathscr{F}(\mathbb{R})}$, $R = \left\{ \frac{\uparrow^{\mathsf{FCD}(\mathbb{R},\mathbb{R})} (]\epsilon;+\infty[\times\mathbb{R})}{\epsilon\in\mathbb{R},\epsilon>0} \right\}$. Then $\bigsqcup R = \uparrow^{\mathsf{FCD}(\mathbb{R},\mathbb{R})} (]0;+\infty[\times\mathbb{R})$. So $\mathcal{X} \left[\bigsqcup R \right] \mathcal{Y}$ and $\forall f \in R : \neg (\mathcal{X} [f] \mathcal{Y})$.

2°. With the same $\mathcal{X}$ and R we have $\langle \bigsqcup R \rangle \mathcal{X} = \top^{\mathscr{F}(\mathbb{R})}$ and $\langle f \rangle \mathcal{X} = \bot^{\mathscr{F}(\mathbb{R})}$ for every $f \in R$, thus $\bigsqcup \left\{ \frac{\langle f \rangle \mathcal{X}}{f \in R} \right\} = \bot^{\mathscr{F}(\mathbb{R})}$.

□

EXAMPLE 1337. $\bigsqcup_{\mathcal{B} \in T} (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}) \neq \mathcal{A} \times^{\mathsf{RLD}} \bigsqcup T$ for some filter $\mathcal{A}$ and set of filters T (with a common base).