Thus

$$f = f \sqcap (\uparrow^{\mathsf{RLD}(\operatorname{Src} f, \operatorname{Dst} f)} G)|_{\operatorname{dom} f} = (\uparrow^{\mathsf{RLD}(\operatorname{Src} f, \operatorname{Dst} f)} F)|_{\operatorname{dom} f} \sqcap (\uparrow^{\mathsf{RLD}(\operatorname{Src} f, \operatorname{Dst} f)} G)|_{\operatorname{dom} f} = (\uparrow^{\mathsf{RLD}(\operatorname{Src} f, \operatorname{Dst} f)} (F \sqcap G))|_{\operatorname{dom} f}.$$

Obviously $F \sqcap G$ is an injection. □

THEOREM 1326. If a reloid f is monovalued and $\operatorname{dom} f$ is an principal filter then f is principal.

PROOF. f is a restricted principal monovalued reloid. Thus $f = F|_{\operatorname{dom} f}$ where F is a principal monovalued reloid. Thus f is principal. □

LEMMA 1327. If a filter $\mathcal{A}$ is isomorphic to a filter $\mathcal{B}$ then if X is a typed set then there exists a typed set Y such that $\uparrow^{\operatorname{Base}(\mathcal{A})} X \sqcap \mathcal{A}$ is a filter isomorphic to $\uparrow^{\operatorname{Base}(\mathcal{B})} Y \sqcap \mathcal{B}$.

PROOF. Let f be a monovalued injective reloid such that $\operatorname{dom} f = \mathcal{A}$, $\operatorname{im} f = \mathcal{B}$. By proposition 626 we have: $\uparrow^{\operatorname{Base}(\mathcal{A})} X \sqcap \mathcal{A} = \mathcal{X}$ where $\mathcal{X}$ is a filter complementive to $\mathcal{A}$. Let $\mathcal{Y} = \mathcal{A} \setminus \mathcal{X}$.

$\langle(\mathsf{FCD})f\rangle\mathcal{X} \sqcap \langle(\mathsf{FCD})f\rangle\mathcal{Y} = \langle(\mathsf{FCD})f\rangle(\mathcal{X} \sqcap \mathcal{Y}) = \bot$ by injectivity of f.

$\langle(\mathsf{FCD})f\rangle\mathcal{X} \sqcup \langle(\mathsf{FCD})f\rangle\mathcal{Y} = \langle(\mathsf{FCD})f\rangle(\mathcal{X} \sqcup \mathcal{Y}) = \langle(\mathsf{FCD})f\rangle\mathcal{A} = \mathcal{B}$. So $\langle(\mathsf{FCD})f\rangle\mathcal{X}$ is a filter complementive to $\mathcal{B}$. So by proposition 626 there exists a set Y such that $\langle(\mathsf{FCD})f\rangle\mathcal{X} = \uparrow Y \sqcap \mathcal{B}$.

$f|_{\mathcal{X}}$ is obviously a monovalued injective reloid with $\operatorname{dom}(f|_{\mathcal{X}}) = \uparrow X \sqcap \mathcal{A}$ and $\operatorname{im}(f|_{\mathcal{X}}) = \uparrow Y \sqcap \mathcal{B}$. So $\uparrow X \sqcap \mathcal{A}$ is isomorphic to $\uparrow Y \sqcap \mathcal{B}$. □

EXAMPLE 1328. $\mathcal{A} \geq_2 \mathcal{B} \wedge \mathcal{B} \geq_2 \mathcal{A}$ but $\mathcal{A}$ is not isomorphic to $\mathcal{B}$ for some filters $\mathcal{A}$ and $\mathcal{B}$.

PROOF. (proof idea by ANDREAS BLASS, rewritten using reloids by me)

Let u_n, h_n with n ranging over the set $\mathbb{Z}$ be sequences of ultrafilters on $\mathbb{N}$ and functions $\mathbb{N} \to \mathbb{N}$ such that $\langle\uparrow^{\mathsf{FCD}(\mathbb{N},\mathbb{N})} h_n\rangle u_{n+1} = u_n$ and u_n are pairwise non-isomorphic. (See [6] for a proof that such ultrafilters and functions exist.)

$\mathcal{A} \stackrel{\text{def}}{=} \bigsqcup_{n\in\mathbb{Z}}(\uparrow^{\mathbb{Z}} \{n\} \times^{\mathsf{RLD}} u_{2n+1})$; $\mathcal{B} \stackrel{\text{def}}{=} \bigsqcup_{n\in\mathbb{Z}}(\uparrow^{\mathbb{Z}} \{n\} \times^{\mathsf{RLD}} u_{2n})$.

Let the **Set**-morphisms $f, g : \mathbb{Z} \times \mathbb{N} \to \mathbb{Z} \times \mathbb{N}$ be defined by the formulas $f(n, x) = (n, h_{2n}x)$ and $g(n, x) = (n - 1, h_{2n-1}x)$.