Proposition 1318. A filter isomorphic to a non-trivial ultrafilter is a non-trivial ultrafilter.

Proof. Let a be a non-trivial ultrafilter and a be isomorphic to b. Then $a \geq_2 b$ and thus b is an ultrafilter. The filter b cannot be trivial because otherwise a would be also trivial. □

Theorem 1319. For an infinite set U there exist $2^{2^{\operatorname{card} U}}$ equivalence classes of isomorphic ultrafilters.

Proof. The number of bijections between any two given subsets of U is no more than $(\operatorname{card} U)^{\operatorname{card} U} = 2^{\operatorname{card} U}$. The number of bijections between all pairs of subsets of U is no more than $2^{\operatorname{card} U} \cdot 2^{\operatorname{card} U} = 2^{\operatorname{card} U}$. Therefore each isomorphism class contains at most $2^{\operatorname{card} U}$ ultrafilters. But there are $2^{2^{\operatorname{card} U}}$ ultrafilters. So there are $2^{2^{\operatorname{card} U}}$ classes. □

Remark 1320. One of the above mentioned equivalence classes contains trivial ultrafilters.

Corollary 1321. There exist non-isomorphic nontrivial ultrafilters on any infinite set.

14.3. Consequences

Theorem 1322. The graph of reloid $\mathcal{F} \times^{\mathsf{RLD}} \uparrow^A \{a\}$ is isomorphic to the filter $\mathcal{F}$ for every set A and $a \in A$.

Proof. From 1309. □

Theorem 1323. If f, g are reloids, $f \sqsubseteq g$ and g is monovalued then $g|_{\operatorname{dom} f} = f$.

Proof. It's simple to show that $f = \bigsqcup \left\{ \frac{f|_a}{a \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Src} f)}} \right\}$ (use the fact that $k \sqsubseteq f|_a$ for some $a \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Src} f)}$ for every $k \in \operatorname{atoms} f$ and the fact that $\mathsf{RLD}(\operatorname{Src} f, \operatorname{Dst} f)$ is atomistic).

Suppose that $g|_{\operatorname{dom} f} \neq f$. Then there exists $a \in \operatorname{atoms} \operatorname{dom} f$ such that $g|_a \neq f|_a$.

Obviously $g|_a \sqsupseteq f|_a$.

If $g|_a \sqsupset f|_a$ then $g|_a$ is not atomic (because $f|_a \neq \bot^{\mathsf{RLD}(\operatorname{Src} f, \operatorname{Dst} f)}$) what contradicts to a theorem above. So $g|_a = f|_a$ what is a contradiction and thus $g|_{\operatorname{dom} f} = f$. □

Corollary 1324. Every monovalued reloid is a restricted principal monovalued reloid.

Proof. Let f be a monovalued reloid. Then there exists a function $F \in \operatorname{GR} f$. So we have

$$(\uparrow^{\mathsf{RLD}(\operatorname{Src} f, \operatorname{Dst} f)} F)|_{\operatorname{dom} f} = f.$$

□

Corollary 1325. Every monovalued injective reloid is a restricted injective monovalued principal reloid.

Proof. Let f be a monovalued injective reloid. There exists a function F such that $f = (\uparrow^{\mathsf{RLD}(\operatorname{Src} f, \operatorname{Dst} f)} F)|_{\operatorname{dom} f}$. Also there exists an injection $G \in \operatorname{up} f$.