LEMMA 1299. $A \notin \mu$.

PROOF. Suppose $A \in \mu$.

Since $A \in \mu$ we have $B_0 \in \mu$ or $B_1 \in \mu$.

So either $B_0 \cap \langle f \rangle^* B_0 \subseteq B_2$ or $B_1 \cap \langle f \rangle^* B_1 \subseteq B_2$. As such by the lemma 1295 we have $B_2 \in \mu$. This is incompatible with $B_2 \cap \langle f \rangle^* B_2 = \emptyset$. So we got a contradiction. □

Let C be the set of points x which are not periodic but $f^n(x)$ is periodic for some positive n.

LEMMA 1300. $C \notin \mu$.

PROOF. Let β be a function $C \to \mathbb{N}$ such that $\beta(x)$ is the least $n \in \mathbb{N}$ such that $f^n(x)$ is periodic.

Let $C_0 = \left\{ \frac{x \in C}{\beta(x) \text{ is even}} \right\}$ and $C_1 = \left\{ \frac{x \in C}{\beta(x) \text{ is odd}} \right\}$.

Obviously $C_j \cap \langle f \rangle^* C_j = \emptyset$ for $j = 0, 1$. Hence by lemma 1295 we have $C_0, C_1 \notin \mu$ and thus $C = C_0 \cup C_1 \notin \mu$. □

Let E be the set of $x \in I$ such that for no $n \in \mathbb{N}$ we have $f^n(x)$ periodic.

LEMMA 1301. Let $x, y \in E$ be such that $f^i(x) = f^j(y)$ and $f^{i'}(x) = f^{j'}(y)$ for some $i, j, i', j' \in \mathbb{N}$. Then $i - j = i' - j'$.

PROOF. $i \mapsto f^i(x)$ is a bijection.

So $y = f^{i-j}(y)$ and $y = f^{i'-j'}(y)$. Thus $f^{i-j}(y) = f^{i'-j'}(y)$ and so $i - j = i' - j'$. □

LEMMA 1302. $E \notin \mu$.

PROOF. Let $D' \subseteq E$ be a subset of E with exactly one element from each equivalence class of the relation $\sim$ on E.

Define the function $\gamma : E \to \mathbb{Z}$ as follows. Let $x \in E$. Let y be the unique element of D' such that $x \sim y$. Choose $i, j \in \mathbb{N}$ such that $f^i(y) = f^j(x)$. Let $\gamma(x) = i - j$. By the last lemma, γ is well-defined.

It is clear that if $x \in E$ then $f(x) \in E$ and moreover $\gamma(f(x)) = \gamma(x) + 1$.

Let $E_0 = \left\{ \frac{x \in E}{\gamma(x) \text{ is even}} \right\}$ and $E_1 = \left\{ \frac{x \in E}{\gamma(x) \text{ is odd}} \right\}$.

We have $E_0 \cap \langle f \rangle^* E_0 = \emptyset \notin \mu$ and hence $E_0 \notin \mu$.

Similarly $E_1 \notin \mu$.

Thus $E = E_0 \cup E_1 \notin \mu$. □

LEMMA 1303. f is the identity function on a set in μ.

PROOF. We have shown $A, C, E \notin \mu$. But the points which lie in none of these sets are exactly points periodic with period 1 that is fixed points of f. Thus the set of fixed points of f belongs to the filter μ. □

14.1.1.2. *The main theorem and its consequences.*

THEOREM 1304. For every ultrafilter a the morphism $(a, a, \mathrm{id}_a^{\mathsf{FCD}})$ is the only

1°. monovalued morphism of the category of reloid triples from a to a;

2°. injective morphism of the category of reloid triples from a to a;

3°. bijective morphism of the category of reloid triples from a to a.

PROOF. We will prove only 1° because the rest follow from it.

Let f be a monovalued morphism of reloid triples from a to a. Then it exists a **Set**-morphism F such that $F \in f$. Trivially $\langle \uparrow^{\mathsf{FCD}} F \rangle a \sqsupseteq a$ and thus $\langle F \rangle^* A \in a$ for every $A \in a$. Thus by the lemma we have that F is the identity function on a set in a and so obviously f is an identity. □