PROPOSITION 1282. For ultrafilters $\geq_2$ is the same as Rudin-Keisler ordering (as defined in [40]).

PROOF. $x \geq_2 y$ iff there exist sets $A \in x$ and $B \in y$ and a bijective **Set**-morphism $f : X \to Y$ such that

$$y \div B = \left\{ \frac{C \in \mathscr{P}Y}{\langle f^{-1} \rangle^* C \in x \div A} \right\}$$

that is when $C \in y \div B \Leftrightarrow \langle f^{-1} \rangle^* C \in x \div A$ what is equivalent to $C \in y \Leftrightarrow \langle f^{-1} \rangle^* C \in x$ what is the definition of Rudin-Keisler ordering. □

REMARK 1283. The relation of being isomorphic for ultrafilters is traditionally called *Rudin-Keisler equivalence*.

OBVIOUS 1284. $(\geq_1) \supseteq (\geq_2)$.

DEFINITION 1285. Let Q and R be binary relations on the set of (small) filters. I will denote $\mathbf{MonRld}_{Q,R}$ the directed multigraph with objects being filters and morphisms such monovalued reloids f that $(\operatorname{dom} f)\ Q\ \mathcal{A}$ and $(\operatorname{im} f)\ R\ \mathcal{B}$.

I will also denote $\mathbf{CoMonRld}_{Q,R}$ the directed multigraph with objects being filters and morphisms such injective reloids f that $(\operatorname{im} f)\ Q\ \mathcal{A}$ and $(\operatorname{dom} f)\ R\ \mathcal{B}$. These are essentially the duals.

Some of these directed multigraphs are categories with reloid composition (see below). By abuse of notation I will denote these categories the same as these directed multigraphs.

LEMMA 1286. $\mathbf{CoMonRld}_{Q,R} \neq \emptyset \Leftrightarrow \mathbf{MonRld}_{Q,R} \neq \emptyset$.

PROOF. $f \in \mathbf{CoMonRld}_{Q,R} \Leftrightarrow (\operatorname{im} f)\ Q\ \mathcal{A} \wedge (\operatorname{dom} f)\ R\ \mathcal{B} \Leftrightarrow (\operatorname{dom} f^{-1})\ Q\ \mathcal{A} \wedge (\operatorname{im} f^{-1})\ R\ \mathcal{B} \Leftrightarrow f^{-1} \in \mathbf{MonRld}_{Q,R}$ for every monovalued reloid f (or what is the same, injective reloid f^{-1}). □

THEOREM 1287. For every filters $\mathcal{A}$ and $\mathcal{B}$ the following are equivalent:

1°. $\mathcal{A} \geq_1 \mathcal{B}$.
2°. $\operatorname{Hom}_{\mathbf{MonRld}_{=,\sqsupseteq}}(\mathcal{A}, \mathcal{B}) \neq \emptyset$.
3°. $\operatorname{Hom}_{\mathbf{MonRld}_{\sqsubseteq,\sqsupseteq}}(\mathcal{A}, \mathcal{B}) \neq \emptyset$.
4°. $\operatorname{Hom}_{\mathbf{MonRld}_{\sqsubseteq,=}}(\mathcal{A}, \mathcal{B}) \neq \emptyset$.
5°. $\operatorname{Hom}_{\mathbf{CoMonRld}_{=,\sqsupseteq}}(\mathcal{A}, \mathcal{B}) \neq \emptyset$.
6°. $\operatorname{Hom}_{\mathbf{CoMonRld}_{\sqsubseteq,\sqsupseteq}}(\mathcal{A}, \mathcal{B}) \neq \emptyset$.
7°. $\operatorname{Hom}_{\mathbf{CoMonRld}_{\sqsubseteq,=}}(\mathcal{A}, \mathcal{B}) \neq \emptyset$.

PROOF.

1°⇒2°. There exists a **Set**-morphism $f : \operatorname{Base}(\mathcal{A}) \to \operatorname{Base}(\mathcal{B})$ such that $\mathcal{B} \sqsubseteq \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A}$. We have

$$\operatorname{dom}(\uparrow^{\mathsf{RLD}} f)|_{\mathcal{A}} = \mathcal{A} \sqcap \top(\operatorname{Base}(\mathcal{A})) = \mathcal{A}$$

and

$$\operatorname{im}(\uparrow^{\mathsf{RLD}} f)|_{\mathcal{A}} = \operatorname{im}(\mathsf{FCD})(\uparrow^{\mathsf{RLD}} f)|_{\mathcal{A}} = \operatorname{im}(\uparrow^{\mathsf{FCD}} f)|_{\mathcal{A}} = \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A} \sqsupseteq \mathcal{B}.$$

Thus $(\uparrow^{\mathsf{RLD}} f)|_{\mathcal{A}}$ is a monovalued reloid such that $\operatorname{dom}(\uparrow^{\mathsf{RLD}} f)|_{\mathcal{A}} = \mathcal{A}$ and $\operatorname{im}(\uparrow^{\mathsf{RLD}} f)|_{\mathcal{A}} \sqsupseteq \mathcal{B}$.

2°⇒3°, 4°⇒3°, 5°⇒6°, 7°⇒6°. Obvious.

3°⇒1°. We have $\mathcal{B} \sqsubseteq \langle (\mathsf{FCD}) f \rangle \mathcal{A}$ for a monovalued reloid $f \in \mathsf{RLD}(\operatorname{Base}(\mathcal{A}), \operatorname{Base}(\mathcal{B}))$. Then there exists a **Set**-morphism $F : \operatorname{Base}(\mathcal{A}) \to \operatorname{Base}(\mathcal{B})$ such that $\mathcal{B} \sqsubseteq \langle \uparrow^{\mathsf{FCD}} F \rangle \mathcal{A}$ that is $\mathcal{A} \geq_1 \mathcal{B}$.