of these directed multigraphs with added composition of morphisms (of directed multigraphs edges). As such I will call vertices of these multigraphs objects and edges morphisms.

DEFINITION 1258. I will denote $\mathbf{GreFunc}_1$ the multigraph whose objects are filters and whose morphisms between objects $\mathcal{A}$ and $\mathcal{B}$ are **Set**-morphisms from $\operatorname{Base}(\mathcal{A})$ to $\operatorname{Base}(\mathcal{B})$ such that $\mathcal{B} \sqsubseteq \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A}$.

DEFINITION 1259. I will denote $\mathbf{GreFunc}_2$ the multigraph whose objects are filters and whose morphisms between objects $\mathcal{A}$ and $\mathcal{B}$ are **Set**-morphisms from $\operatorname{Base}(\mathcal{A})$ to $\operatorname{Base}(\mathcal{B})$ such that $\mathcal{B} = \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A}$.

DEFINITION 1260. Let $\mathcal{A}$ be a filter on a set X and $\mathcal{B}$ be a filter on a set Y. $\mathcal{A} \geq_1 \mathcal{B}$ iff $\operatorname{Hom}_{\mathbf{GreFunc}_1}(\mathcal{A}, \mathcal{B})$ is not empty.

DEFINITION 1261. Let $\mathcal{A}$ be a filter on a set X and $\mathcal{B}$ be a filter on a set Y. $\mathcal{A} \geq_2 \mathcal{B}$ iff $\operatorname{Hom}_{\mathbf{GreFunc}_2}(\mathcal{A}, \mathcal{B})$ is not empty.

PROPOSITION 1262.

1°. $f \in \operatorname{Hom}_{\mathbf{GreFunc}_1}(\mathcal{A}, \mathcal{B})$ iff f is a **Set**-morphism from $\operatorname{Base}(\mathcal{A})$ to $\operatorname{Base}(\mathcal{B})$ such that
$$C \in \mathcal{B} \Leftarrow \langle f^{-1} \rangle^* C \in \mathcal{A}$$
for every $C \in \mathscr{P} \operatorname{Base}(\mathcal{B})$.

2°. $f \in \operatorname{Hom}_{\mathbf{GreFunc}_2}(\mathcal{A}, \mathcal{B})$ iff f is a **Set**-morphism from $\operatorname{Base}(\mathcal{A})$ to $\operatorname{Base}(\mathcal{B})$ such that
$$C \in \mathcal{B} \Leftrightarrow \langle f^{-1} \rangle^* C \in \mathcal{A}$$
for every $C \in \mathscr{P} \operatorname{Base}(\mathcal{B})$.

PROOF.

1°.

$$f \in \operatorname{Hom}_{\mathbf{GreFunc}_1}(\mathcal{A}, \mathcal{B}) \Leftrightarrow \mathcal{B} \sqsubseteq \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A} \Leftrightarrow \forall C \in \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A} : C \in \mathcal{B} \Leftrightarrow \forall C \in \mathscr{P} \operatorname{Base}(\mathcal{B}) : (\langle f^{-1} \rangle^* C \in \mathcal{A} \Rightarrow C \in \mathcal{B}).$$

2°.

$$f \in \operatorname{Hom}_{\mathbf{GreFunc}_2}(\mathcal{A}, \mathcal{B}) \Leftrightarrow \mathcal{B} = \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A} \Leftrightarrow \forall C : (C \in \mathcal{B} \Leftrightarrow C \in \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A}) \Leftrightarrow \forall C \in \mathscr{P} \operatorname{Base}(\mathcal{B}) : (C \in \mathcal{B} \Leftrightarrow C \in \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A}) \Leftrightarrow \forall C \in \mathscr{P} \operatorname{Base}(\mathcal{B}) : (\langle f^{-1} \rangle^* C \in \mathcal{A} \Leftrightarrow C \in \mathcal{B}).$$

□

DEFINITION 1263. The directed multigraph **FuncBij** is the directed multigraph got from $\mathbf{GreFunc}_2$ by restricting to only bijective morphisms.

DEFINITION 1264. A filter $\mathcal{A}$ is *directly isomorphic* to a filter $\mathcal{B}$ iff there is a morphism $f \in \operatorname{Hom}_{\mathbf{FuncBij}}(\mathcal{A}, \mathcal{B})$.

OBVIOUS 1265. $f \in \operatorname{Hom}_{\mathbf{GreFunc}_1}(\mathcal{A}, \mathcal{B}) \Leftrightarrow \mathcal{B} \sqsubseteq \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A}$ for every **Set**-morphism from $\operatorname{Base}(\mathcal{A})$ to $\operatorname{Base}(\mathcal{B})$.

OBVIOUS 1266. $f \in \operatorname{Hom}_{\mathbf{GreFunc}_2}(\mathcal{A}, \mathcal{B}) \Leftrightarrow \mathcal{B} = \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A}$ for every **Set**-morphism from $\operatorname{Base}(\mathcal{A})$ to $\operatorname{Base}(\mathcal{B})$.

COROLLARY 1267. $\mathcal{A} \geq_1 \mathcal{B}$ iff it exists a **Set**-morphism $f : \operatorname{Base}(\mathcal{A}) \to \operatorname{Base}(\mathcal{B})$ such that $\mathcal{B} \sqsubseteq \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A}$.

COROLLARY 1268. $\mathcal{A} \geq_2 \mathcal{B}$ iff it exists a **Set**-morphism $f : \operatorname{Base}(\mathcal{A}) \to \operatorname{Base}(\mathcal{B})$ such that $\mathcal{B} = \langle \uparrow^{\mathsf{FCD}} f \rangle \mathcal{A}$.