

To prove that  $\Gamma$  is not  $\alpha$ -thick it's enough to prove that every set  $A$  such that  $A \times A \subseteq \Gamma$  is finite.

Suppose for the contrary that  $A$  is infinite. Then  $A$  contains more than one non-zero points  $y, z$  ( $y \neq z$ ). Without loss of generality  $y < z$ . So we have that  $(y, z)$  is not of the form  $(y, y)$  nor  $(0, y)$  nor  $(y, 0)$ . Therefore  $A \times A$  isn't a subset of  $\Gamma$ .  $\square$

### 13.2. Totally bounded endoreloids

The below is a straightforward generalization of the customary definition of totally bounded sets on uniform spaces (it's proved below that for uniform spaces the below definitions are equivalent).

DEFINITION 1239. An endoreloid  $f$  is  $\alpha$ -totally bounded ( $\text{totBound}_\alpha(f)$ ) if every  $E \in \text{up } f$  is  $\alpha$ -thick.

DEFINITION 1240. An endoreloid  $f$  is  $\beta$ -totally bounded ( $\text{totBound}_\beta(f)$ ) if every  $E \in \text{up } f$  is  $\beta$ -thick.

REMARK 1241. We could rewrite the above definitions in a more algebraic way like  $\text{up } f \subseteq \text{thick}_\alpha$  (with  $\text{thick}_\alpha$  would be defined as a set rather than as a predicate), but we don't really need this simplification.

PROPOSITION 1242. If an endoreloid is  $\alpha$ -totally bounded then it is  $\beta$ -totally bounded.

PROOF. Because  $\text{thick}_\alpha(E) \Rightarrow \text{thick}_\beta(E)$ .  $\square$

PROPOSITION 1243. If an endoreloid  $f$  is reflexive and  $\text{Ob } f$  is finite then  $f$  is both  $\alpha$ -totally bounded and  $\beta$ -totally bounded.

PROOF. It enough to prove that  $f$  is  $\alpha$ -totally bounded. Really, every  $E \in \text{up } f$  is reflexive. Thus  $\{x\} \times \{x\} \subseteq \text{GR } E$  for  $x \in \text{Ob } f$  and thus  $\left\{ \frac{\{x\}}{x \in \text{Ob } f} \right\}$  is a sought for finite cover of  $\text{Ob } f$ .  $\square$

OBVIOUS 1244.

- A principal endoreloid induced by a **Rel**-morphism  $E$  is  $\alpha$ -totally bounded iff  $E$  is  $\alpha$ -thick.
- A principal endoreloid induced by a **Rel**-morphism  $E$  is  $\beta$ -totally bounded iff  $E$  is  $\beta$ -thick.

EXAMPLE 1245. There is a  $\beta$ -totally bounded endoreloid which is not  $\alpha$ -totally bounded.

PROOF. It follows from the example above and properties of principal endoreloids.  $\square$

### 13.3. Special case of uniform spaces

Remember that *uniform space* is essentially the same as symmetric, reflexive and transitive endoreloid.

THEOREM 1246. Let  $f$  be such an endoreloid that  $f \circ f^{-1} \subseteq f$ . Then  $f$  is  $\alpha$ -totally bounded iff it is  $\beta$ -totally bounded.

PROOF.

$\Rightarrow$ . Proved above.