OBVIOUS 1213. A filter $\mathcal{A}$ is connected regarding a reloid μ iff it is connected regarding the reloid $\mu \sqcap (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{A})$.

OBVIOUS 1214. A filter $\mathcal{A}$ is connected regarding a funcoid μ iff it is connected regarding the funcoid $\mu \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{A})$.

THEOREM 1215. A filter $\mathcal{A}$ is connected regarding a reloid f iff $\mathcal{A}$ is connected regarding every $F \in \left\langle \uparrow^{\mathsf{RLD}} \right\rangle^* \operatorname{up} f$.

PROOF.

$\Rightarrow$. Obvious.

$\Leftarrow$. $\mathcal{A}$ is connected regarding $\uparrow^{\mathsf{RLD}} F$ iff $S_1(F) = F^1 \sqcup F^2 \sqcup \cdots \in \operatorname{up}(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{A})$.

$$S_1^*(f) = \prod^{\mathsf{RLD}}_{F \in \operatorname{up} f} S_1(F) \sqsupseteq \prod_{F \in \operatorname{up} f} (\mathcal{A} \times^{\mathsf{RLD}} \mathcal{A}) = \mathcal{A} \times^{\mathsf{RLD}} \mathcal{A}.$$

□

CONJECTURE 1216. A filter $\mathcal{A}$ is connected regarding a funcoid f iff $\mathcal{A}$ is connected regarding every $F \in \left\langle \uparrow^{\mathsf{FCD}} \right\rangle^* \operatorname{up} f$.

The above conjecture is open even for the case when $\mathcal{A}$ is a principal filter.

CONJECTURE 1217. A filter $\mathcal{A}$ is connected regarding a reloid f iff it is connected regarding the funcoid $(\mathsf{FCD})f$.

The above conjecture is true in the special case of principal filters:

PROPOSITION 1218. A filter $\uparrow A$ (for a typed set A) is connected regarding an endoreloid f on the suitable object iff it is connected regarding the endofuncoid $(\mathsf{FCD})f$.

PROOF. $\uparrow A$ is connected regarding a reloid f iff A is connected regarding every $F \in \operatorname{up} f$ that is when (taken into account that connectedness for $\uparrow^{\mathsf{RLD}} F$ is the same as connectedness of $\uparrow^{\mathsf{FCD}} F$)

$$\begin{aligned}
\forall F \in \operatorname{up} f \forall \mathcal{X}, \mathcal{Y} \in \mathscr{F}(\operatorname{Ob} f) \setminus \{\bot^{\mathscr{F}(\operatorname{Ob} f)}\} : (\mathcal{X} \sqcup \mathcal{Y} = \uparrow A \Rightarrow \mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y}) \Leftrightarrow \\
\forall \mathcal{X}, \mathcal{Y} \in \mathscr{F}(\operatorname{Ob} f) \setminus \{\bot^{\mathscr{F}(\operatorname{Ob} f)}\} \forall F \in \operatorname{up} f : (\mathcal{X} \sqcup \mathcal{Y} = \uparrow A \Rightarrow \mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y}) \Leftrightarrow \\
\forall \mathcal{X}, \mathcal{Y} \in \mathscr{F}(\operatorname{Ob} f) \setminus \{\bot^{\mathscr{F}(\operatorname{Ob} f)}\} (\mathcal{X} \sqcup \mathcal{Y} = \uparrow A \Rightarrow \forall F \in \operatorname{up} f : \mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y}) \Leftrightarrow \\
\forall \mathcal{X}, \mathcal{Y} \in \mathscr{F}(\operatorname{Ob} f) \setminus \{\bot^{\mathscr{F}(\operatorname{Ob} f)}\} (\mathcal{X} \sqcup \mathcal{Y} = \uparrow A \Rightarrow \mathcal{X} \left[(\mathsf{FCD})f\right] \mathcal{Y})
\end{aligned}$$

that is when the set $\uparrow A$ is connected regarding the funcoid $(\mathsf{FCD})f$. □

CONJECTURE 1219. A set A is connected regarding an endofuncoid μ iff for every $a, b \in A$ there exists a totally ordered set $P \subseteq A$ such that $\min P = a$, $\max P = b$ and

$$\forall q \in P \setminus \{b\} : \left\{ \frac{x \in P}{x \leq q} \right\} [\mu]^* \left\{ \frac{x \in P}{x > q} \right\}.$$

Weaker condition:

$$\forall q \in P \setminus \{b\} : \left\{ \frac{x \in P}{x \leq q} \right\} [\mu]^* \left\{ \frac{x \in P}{x > q} \right\} \vee \forall q \in P \setminus \{a\} : \left\{ \frac{x \in P}{x < q} \right\} [\mu]^* \left\{ \frac{x \in P}{x \geq q} \right\}.$$

12.5. Algebraic properties of S and S^*

THEOREM 1220. $S^*(S^*(f)) = S^*(f)$ for every endoreloid f.