CHAPTER 12

Connectedness regarding funcoids and reloids

12.1. Some lemmas

LEMMA 1185. Let U be a set, $A, B \in \mathscr{T}U$ be typed sets, f be an endo-funcoid on U. If $\neg(A\ [f]^*\ B) \wedge A \sqcup B \in \operatorname{up}(\operatorname{dom} f \sqcup \operatorname{im} f)$ then f is closed on A.

PROOF. Let $A \sqcup B \in \operatorname{up}(\operatorname{dom} f \sqcup \operatorname{im} f)$.

$$\begin{aligned} \neg(A\ [f]^*\ B) &\Leftrightarrow \\ B \sqcap \langle f\rangle^* A = \bot &\Rightarrow \\ (\operatorname{dom} f \sqcup \operatorname{im} f) \sqcap B \sqcap \langle f\rangle^* A = \bot &\Rightarrow \\ ((\operatorname{dom} f \sqcup \operatorname{im} f) \setminus A) \sqcap \langle f\rangle^* A = \bot &\Leftrightarrow \\ \langle f\rangle^* A \sqsubseteq A. \end{aligned}$$

□

COROLLARY 1186. If $\neg(A\ [f]^*\ B) \wedge A \sqcup B \in \operatorname{up}(\operatorname{dom} f \sqcup \operatorname{im} f)$ then f is closed on $A \setminus B$ for a funcoid $f \in \mathsf{FCD}(U, U)$ for every sets U and typed sets $A, B \in \mathscr{T}U$.

PROOF. Let $\neg(A\ [f]^*\ B) \wedge A \sqcup B \in \operatorname{up}(\operatorname{dom} f \sqcup \operatorname{im} f)$. Then

$$\neg((A \setminus B)\ [f]^*\ B) \wedge (A \setminus B) \sqcup B \in \operatorname{up}(\operatorname{dom} f \sqcup \operatorname{im} f).$$

□

LEMMA 1187. If $\neg(A\ [f]^*\ B) \wedge A \sqcup B \in \operatorname{up}(\operatorname{dom} f \sqcup \operatorname{im} f)$ then $\neg(A\ [f^n]^*\ B)$ for every whole positive n.

PROOF. Let $\neg(A\ [f]^*\ B) \wedge A \sqcup B \in \operatorname{up}(\operatorname{dom} f \sqcup \operatorname{im} f)$. From the above lemma $\langle f\rangle^* A \sqsubseteq A$. $B \sqcap \langle f\rangle A = \bot$, consequently $\langle f\rangle^* A \sqsubseteq A \setminus B$. Because (by the above corollary) f is closed on $A \setminus B$, then $\langle f\rangle\langle f\rangle A \sqsubseteq A \setminus B$, $\langle f\rangle\langle f\rangle\langle f\rangle A \sqsubseteq A \setminus B$, etc. So $\langle f^n\rangle A \sqsubseteq A \setminus B$, $B \asymp \langle f^n\rangle A$, $\neg(A\ [f^n]^*\ B)$. □

12.2. Endomorphism series

DEFINITION 1188. $S_1(\mu) = \mu \sqcup \mu^2 \sqcup \mu^3 \sqcup \ldots$ for an endomorphism μ of a precategory with countable join of morphisms (that is join defined for every countable set of morphisms).

DEFINITION 1189. $S(\mu) = \mu^0 \sqcup S_1(\mu) = \mu^0 \sqcup \mu \sqcup \mu^2 \sqcup \mu^3 \sqcup \ldots$where $\mu^0 = 1_{\operatorname{Ob}\mu}$ (identity morphism for the object $\operatorname{Ob}\mu$) where $\operatorname{Ob}\mu$ is the object of endomorphism μ for an endomorphism μ of a category with countable join of morphisms.

I call S_1 and S *endomorphism series*.

PROPOSITION 1190. The relation $S(\mu)$ is transitive for the category **Rel**.