2°⇒3°. Let B be a neighborhood of $f(x)$. Then there is an open neighborhood $B' \sqsubseteq B$ of $f(x)$. $\langle f^{-1}\rangle^* B'$ is open and thus is a neighborhood of x ($x \in \langle f^{-1}\rangle^* B'$ because $f(x) \in B'$). Consequently $\langle f^{-1}\rangle^* B$ is a neighborhood of x.

Alternative proof of 2°⇔4°: http://math.stackexchange.com/a/1855782/4876

□

11.4. $\mathrm{C}(\mu \circ \mu^{-1}, \nu \circ \nu^{-1})$

PROPOSITION 1181. $f \in \mathrm{C}(\mu, \nu) \Rightarrow f \in \mathrm{C}''(\mu \circ \mu^{-1}, \nu \circ \nu^{-1})$ for endofuncoids μ, ν and monovalued funcoid $f \in \mathsf{FCD}(\mathrm{Ob}\,\mu, \mathrm{Ob}\,\nu)$.

PROOF. Let $f \in \mathrm{C}(\mu, \nu)$.
$X \; [f \circ \mu \circ \mu^{-1} \circ f^{-1}]^* \; Z \Leftrightarrow \exists p \in \operatorname{atoms}^{\mathscr{F}} : \left(X \; [\mu^{-1} \circ f^{-1}]^* \; p \wedge p \; [f \circ \mu]^* \; Z\right) \Leftrightarrow \exists p \in \operatorname{atoms}^{\mathscr{F}} : (p \; [f \circ \mu]^* \; X \wedge p \; [f \circ \mu]^* \; Z) \Rightarrow \exists p \in \operatorname{atoms}^{\mathscr{F}} : (p \; [\nu \circ f]^* \; X \wedge p \; [\nu \circ f]^* \; Z) \Leftrightarrow \exists p \in \operatorname{atoms}^{\mathscr{F}} : \left(\langle f\rangle^* p \; [\nu]^* \; X \wedge \langle f\rangle^* p \; [\nu]^* \; Z\right) \Rightarrow X \; [\nu \circ \nu^{-1}]^* \; Z$ (taken into account monovaluedness of f and thus that $\langle f\rangle^* p$ is atomic or least). Thus $f \circ \mu \circ \mu^{-1} \circ f^{-1} \sqsubseteq \nu \circ \nu^{-1}$ that is $f \in \mathrm{C}''(\mu \circ \mu^{-1}, \nu \circ \nu^{-1})$. □

PROPOSITION 1182. $f \in \mathrm{C}''(\mu \circ \mu^{-1}, \nu \circ \nu^{-1}) \Rightarrow f \in \mathrm{C}''(\mu, \nu)$ for complete endofuncoids μ, ν and principal funcoid $f \in \mathsf{FCD}(\mathrm{Ob}\,\mu, \mathrm{Ob}\,\nu)$, provided that μ is reflexive, and ν is T_1-separable.

PROOF. $f \in \mathrm{C}''(\mu \circ \mu^{-1}, \nu \circ \nu^{-1}) \Leftrightarrow f \circ \mu \circ \mu^{-1} \circ f^{-1} \sqsubseteq \nu \circ \nu^{-1} \Rightarrow$ (reflexivity of μ) $\Rightarrow f \circ \mu \circ f^{-1} \sqsubseteq \nu \circ \nu^{-1} \Leftrightarrow f \circ \mu^{-1} \circ f^{-1} \sqsubseteq \nu \circ \nu^{-1} \Rightarrow \langle f \circ \mu^{-1} \circ f^{-1}\rangle^* X \sqsubseteq \langle \nu\rangle^* \langle \nu^{-1}\rangle^* X \Rightarrow \operatorname{Cor}\left\langle f \circ \mu^{-1} \circ f^{-1}\right\rangle^* X \sqsubseteq \operatorname{Cor}\langle \nu\rangle^* \langle \nu^{-1}\rangle^* X \Leftrightarrow \langle f \circ \mu^{-1} \circ f^{-1}\rangle^* X \sqsubseteq \operatorname{Cor}\langle \nu\rangle^* \langle \nu^{-1}\rangle^* X \Rightarrow$ (T_1-separability) $\Rightarrow \langle f \circ \mu^{-1} \circ f^{-1}\rangle^* X \sqsubseteq \langle \nu^{-1}\rangle^* X$ for any typed set X on $\mathrm{Ob}\,\nu$. Thus $f \in \mathrm{C}''(\mu \circ \mu^{-1}, \nu \circ \nu^{-1}) \Rightarrow f \circ \mu^{-1} \circ f^{-1} \sqsubseteq \nu^{-1} \Leftrightarrow f \circ \mu \circ f^{-1} \sqsubseteq \nu \Leftrightarrow f \in \mathrm{C}''(\mu, \nu)$. □

THEOREM 1183. $f \in \mathrm{C}(\mu \circ \mu^{-1}, \nu \circ \nu^{-1}) \Leftrightarrow f \in \mathrm{C}(\mu, \nu)$ for complete endofuncoids μ, ν and principal monovalued and entirely defined funcoid $f \in \mathsf{FCD}(\mathrm{Ob}\,\mu, \mathrm{Ob}\,\nu)$, provided that μ is reflexive, and ν is T_1-separable.

PROOF. Two above propositions and theorem 1175. □

11.5. Continuity of a restricted morphism

Consider some partially ordered semigroup. (For example it can be the semigroup of funcoids or semigroup of reloids on some set regarding the composition.) Consider also some lattice (*lattice of objects*). (For example take the lattice of set theoretic filters.)

We will map every object A to so called *restricted identity* element I_A of the semigroup (for example restricted identity funcoid or restricted identity reloid). For identity elements we will require

1°. $I_A \circ I_B = I_{A \sqcap B}$;
2°. $f \circ I_A \sqsubseteq f$; $I_A \circ f \sqsubseteq f$.

In the case when our semigroup is "dagger" (that is is a dagger precategory) we will require also $(I_A)^\dagger = I_A$.

We can define restricting an element f of our semigroup to an object A by the formula $f|_A = f \circ I_A$.

We can define *rectangular restricting* an element f of our semigroup to objects A and B as $I_B \circ f \circ I_A$. Optionally we can define direct product $A \times B$ of two objects by the formula (true for funcoids and for reloids):

$$f \sqcap (A \times B) = I_B \circ f \circ I_A.$$