PROOF. $\uparrow^{\mathscr{F}(\operatorname{Src} f\times\operatorname{Dst} g)}\operatorname{dom}(G\otimes F)\sqsupseteq\operatorname{dom}\bigsqcap^{\mathscr{F}(\operatorname{Src} f\times\operatorname{Dst} g)}\left\{\frac{G\otimes F}{F\in\operatorname{GR} f,G\in\operatorname{GR} g}\right\}$. Thus

$$g\circ f\sqsupseteq\operatorname{dom}\bigsqcap^{\mathscr{F}(\operatorname{Src} f\times\operatorname{Dst} g)}\left\{\frac{G\otimes F}{F\in\operatorname{GR} f,G\in\operatorname{GR} g}\right\}.$$

Let $X\in\operatorname{up}\operatorname{dom}\bigsqcap^{\mathscr{F}(\operatorname{Src} f\times\operatorname{Dst} g)}\left\{\frac{G\otimes F}{F\in\operatorname{up} f,G\in\operatorname{up} g}\right\}$. Then there exist Y such that

$$X\times Y\in\operatorname{up}\bigsqcap^{\mathscr{F}(\operatorname{Src} f\times\operatorname{Dst} g)}\left\{\frac{G\otimes F}{F\in\operatorname{up} f,G\in\operatorname{up} g}\right\}.$$

So because it is a generalized filter base $X\times Y\supseteq G\otimes F$ for some $F\in\operatorname{up} f$, $G\in\operatorname{up} g$. Thus $X\in\operatorname{up}\operatorname{dom}(G\otimes F)$. $X\in\operatorname{up}(g\circ f)$. □

We can define $g\otimes f$ for reloids f, g:

$$g\otimes f=\left\{\frac{G\otimes F}{F\in\operatorname{GR} f,G\in\operatorname{GR} g}\right\}.$$

Then

$$g\circ f=\bigsqcap^{\mathscr{F}(\operatorname{Src} f\times\operatorname{Dst} g)}\langle\operatorname{dom}\rangle^*(g\otimes f)=\operatorname{dom}\left\langle\uparrow^{\mathsf{RLD}(\operatorname{Src} f\times\operatorname{Dst} g,\mho)}\right\rangle^*(g\otimes f).$$

10.3. Lemmas for the main result

LEMMA 1158. $(g\otimes f)\cap(h\otimes f)=(g\cap h)\otimes f$ for binary relations f, g, h.

PROOF.

$$(g\cap h)\otimes f=\Theta_0 f\cap\Theta_1(g\cap h)=\Theta_0 f\cap(\Theta_1 g\cap\Theta_1 h)=\\(\Theta_0 f\cap\Theta_1 g)\cap(\Theta_0 f\cap\Theta_1 h)=(g\otimes f)\cap(h\otimes f).$$

□

LEMMA 1159. Let $F=\uparrow^{\mathsf{RLD}} f$ be a principal reloid (for a **Rel**-morphism f), T be a set of reloids from $\operatorname{Dst} F$ to a set V.

$$\bigsqcap_{G\in\operatorname{up}\bigsqcup T}^{\mathsf{RLD}(\operatorname{Src} f\times V,\mho)}(G\otimes f)=\bigsqcup_{G\in T}\bigsqcap^{\mathsf{RLD}(\operatorname{Src} f\times V,\mho)}(G\otimes F).$$

PROOF. $\bigsqcap_{G\in\operatorname{up}\bigsqcup T}^{\mathsf{RLD}(\operatorname{Src} f\times V,\mho)}(G\otimes f)\sqsupseteq\bigsqcup_{G\in T}\bigsqcap^{\mathsf{RLD}(\operatorname{Src} f\times V,\mho)}(G\otimes F)$ is obvious.

Let $K\in\operatorname{up}\bigsqcup_{G\in T}\bigsqcap^{\mathsf{RLD}(\operatorname{Src} f\times V,\mho)}(G\otimes F)$. Then for each $G\in T$

$$K\in\operatorname{up}\bigsqcap^{\mathsf{RLD}(\operatorname{Src} f\times V,\mho)}(G\otimes F);$$

$K\in\operatorname{up}\bigsqcap^{\mathsf{RLD}(\operatorname{Src} f\times V,\mho)}\left\{\frac{\Gamma\otimes f}{\Gamma\in G}\right\}$. Then $K\in\left\{\frac{\Gamma\otimes f}{\Gamma\in G}\right\}$ by properties of generalized filter bases.

$K\in\left\{\frac{(\Gamma_0\cap\dots\cap\Gamma_n)\otimes f}{n\in\mathbb{N},\Gamma_i\in G}\right\}=\left\{\frac{(\Gamma_0\otimes f)\cap\dots\cap(\Gamma_n\otimes f)}{n\in\mathbb{N},\Gamma_i\in G}\right\}$.

$\forall G\in T:K\supseteq(\Gamma_{G,0}\otimes f)\cap\dots\cap(\Gamma_{G,n}\otimes f)$ for some $n\in\mathbb{N}$, $\Gamma_{G,i}\in G$.

$K\supseteq\left\{\frac{(\Gamma_0\otimes f)\cap\dots\cap(\Gamma_n\otimes f)}{n\in\mathbb{N},\Gamma_i\in G}\right\}$ where $\Gamma_i=\bigcup_{g\in G}\Gamma_{g,i}\in\operatorname{up}\bigsqcup T$.