

PROPOSITION 1130. $(\text{RLD})_{\text{in}}f$ is reflexive iff f is reflexive (for every endofunctor f).

PROOF. $(\text{RLD})_{\text{in}}f$ is reflexive iff $(\text{FCD})(\text{RLD})_{\text{in}}f$ is reflexive iff f is reflexive. \square

OBVIOUS 1131. (FCD) , $(\text{RLD})_{\text{in}}$, and $(\text{RLD})_{\text{out}}$ preserve symmetry of the argument functor or reloid.

PROPOSITION 1132. $a \times_F^{\text{RLD}} a = \perp$ for every nontrivial ultrafilter a .

PROOF. $a \times_F^{\text{RLD}} a = (\text{RLD})_{\text{out}}(a \times^{\text{FCD}} a) = \prod^{\text{RLD}} \text{up}(a \times^{\text{FCD}} a) \subseteq 1^{\text{FCD}} \cap (\top^{\text{FCD}} \setminus 1^{\text{FCD}}) = \perp^{\text{FCD}}$. \square

EXAMPLE 1133. There exist filters \mathcal{A} and \mathcal{B} such that $(\text{FCD})(\mathcal{A} \times_F^{\text{RLD}} \mathcal{B}) \subseteq \mathcal{A} \times^{\text{FCD}} \mathcal{B}$.

PROOF. Take $\mathcal{A} = \mathcal{B} = a$ for a nontrivial ultrafilter a . $a \times_F^{\text{RLD}} a = \perp$. Thus $(\text{FCD})(a \times_F^{\text{RLD}} a) = \perp \subseteq a \times^{\text{FCD}} a$. \square

CONJECTURE 1134. There exist filters \mathcal{A} and \mathcal{B} such that $(\text{FCD})(\mathcal{A} \times \mathcal{B}) \subseteq \mathcal{A} \times^{\text{FCD}} \mathcal{B}$.

EXAMPLE 1135. There is such a non-symmetric reloid f that $(\text{FCD})f$ is symmetric.

PROOF. Take $f = ((\text{RLD})_{\text{in}}(=)|_{\mathbb{R}}) \cap (\geq)_{\mathbb{R}}$. f is non-symmetric because $f \not\asymp (>)_{\mathbb{R}}$ but $f \asymp (<)_{\mathbb{R}}$. $(\text{FCD})f = (=)|_{\mathbb{R}}$ because $(=)|_{\mathbb{R}} \subseteq f \subseteq (\text{RLD})_{\text{in}}(=)|_{\mathbb{R}}$. \square

PROPOSITION 1136. If $(\text{RLD})_{\text{in}}f$ is symmetric then endofunctor f is symmetric.

PROOF. Suppose $(\text{RLD})_{\text{in}}f$ is symmetric then $f = (\text{FCD})(\text{RLD})_{\text{in}}f$ is symmetric. \square

CONJECTURE 1137. If $(\text{RLD})_{\text{out}}f$ is symmetric then endofunctor f is symmetric.

PROPOSITION 1138. If f is a transitive endoreloid, then $(\text{FCD})f$ is a transitive functor.

PROOF. $f = f \circ f$; $(\text{FCD})f = (\text{FCD})(f \circ f)$; $(\text{FCD})f = (\text{FCD})f \circ (\text{FCD})f$. \square

CONJECTURE 1139. There exists a non-transitive endoreloid f such that $(\text{FCD})f$ is a transitive functor.

PROPOSITION 1140. $(\text{RLD})_{\text{in}}f$ is transitive iff f is transitive (for every endofunctor f).

PROOF. $f = f \circ f \Rightarrow (\text{RLD})_{\text{in}}f = (\text{RLD})_{\text{in}}(f \circ f) \Leftrightarrow$ (theorem 1118) $\Leftrightarrow (\text{RLD})_{\text{in}}f = (\text{RLD})_{\text{in}}f \circ (\text{RLD})_{\text{in}}f \Rightarrow (\text{FCD})(\text{RLD})_{\text{in}}f = (\text{FCD})(\text{RLD})_{\text{in}}f \circ (\text{FCD})(\text{RLD})_{\text{in}}f \Leftrightarrow f = f \circ f$. Thus $f = f \circ f \Leftrightarrow (\text{RLD})_{\text{in}}f \circ (\text{RLD})_{\text{in}}f$. \square

CONJECTURE 1141.

- 1°. There exists such a transitive endofunctor f , that $(\text{RLD})_{\text{out}}f$ is not a transitive reloid.
- 2°. There exists such a non-transitive endofunctor f , that $(\text{RLD})_{\text{out}}f$ is transitive reloid.