Proof. Really, $g = \bigsqcup_{x \in \operatorname{Src} f}(\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{RLD}} \langle f\rangle^* @\{x\})$ for a complete reloid g and thus

$$(\mathsf{FCD})g = \bigsqcup_{x \in \operatorname{Src} f} (\mathsf{FCD})(\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{RLD}} \langle f\rangle^* @\{x\}) = \bigsqcup_{x \in \operatorname{Src} f} (\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{FCD}} \langle f\rangle^* @\{x\}) = \theta g.$$

□

Lemma 1121. $(\mathsf{RLD})_{\mathrm{out}} f = \theta^{-1} f$ for every complete funcoid f.

Proof. We have $f = \bigsqcup_{x \in \operatorname{Src} f}(\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{FCD}} \langle f\rangle^* @\{x\})$. We need to prove $(\mathsf{RLD})_{\mathrm{out}} f = \bigsqcup_{x \in \operatorname{Src} f}(\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{RLD}} \langle f\rangle^* @\{x\})$.

Really, $(\mathsf{RLD})_{\mathrm{out}} f \sqsupseteq \bigsqcup_{x \in \operatorname{Src} f}(\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{RLD}} \langle f\rangle^* @\{x\})$.

It remains to prove that $\bigsqcup_{x \in \operatorname{Src} f}(\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{RLD}} \langle f\rangle^* @\{x\}) \sqsupseteq (\mathsf{RLD})_{\mathrm{out}} f$.

Let $L \in \operatorname{up} \bigsqcup_{x \in \operatorname{Src} f}(\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{RLD}} \langle f\rangle^* @\{x\})$. We will prove $L \in \operatorname{up}(\mathsf{RLD})_{\mathrm{out}} f$.

We have

$$L \in \bigcap_{x \in \operatorname{Src} f} \operatorname{up}(\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{RLD}} \langle f\rangle^* @\{x\}).$$

$\langle L\rangle^* \{x\} = G(x)$ for some $G(x) \in \operatorname{up}\langle f\rangle^* @\{x\}$ (because $L \in \operatorname{up}(\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{RLD}} \langle f\rangle^* @\{x\})$.

Thus $L = G \in \operatorname{up} f$ (because f is complete). Thus $L \in \operatorname{up} f$ and so $L \in \operatorname{up}(\mathsf{RLD})_{\mathrm{out}} f$.

□

Proposition 1122. (FCD) and $(\mathsf{RLD})_{\mathrm{out}}$ form mutually inverse bijections between complete reloids and complete funcoids.

Proof. From two last lemmas. □

Theorem 1123. The diagram at the figure 1 (with the "unnamed" arrow from $\mathsf{ComplRLD}(A, B)$ to $\mathscr{F}(B)^A$ *defined* as the inverse isomorphism of its opposite arrow) is a commutative diagram (in category **Set**), every arrow in this diagram is an isomorphism. Every cycle in this diagram is an identity (therefore "parallel" arrows are mutually inverse). The arrows preserve order.

Figure 1.