Let $F \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} f$, $G \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} g$. Then $\operatorname{dom} F \times^{\mathsf{FCD}} \operatorname{im} F \sqsubseteq f$ and $\operatorname{dom} G \times^{\mathsf{FCD}} \operatorname{im} G \sqsubseteq g$. Provided that $\operatorname{im} F \not\asymp \operatorname{dom} G$, we have:

$$\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} G = (\operatorname{dom} G \times^{\mathsf{RLD}} \operatorname{im} G) \circ (\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} F) =$$

$$\bigsqcup^{\mathsf{RLD}} \left\{ \frac{G' \circ F'}{F' \in \operatorname{atoms}(\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} F), G' \in \operatorname{atoms}(\operatorname{dom} G \times^{\mathsf{RLD}} \operatorname{im} G)} \right\} \sqsubseteq (*)$$

$$\bigsqcup^{\mathsf{RLD}} \left\{ \frac{G' \circ F'}{F' \in \operatorname{atoms}^{\mathsf{RLD}(\operatorname{Src} F, \operatorname{Dst} F)}, G' \in \operatorname{atoms}^{\mathsf{RLD}(\operatorname{Src} G, \operatorname{Dst} G)}, F' \sqsubseteq (\mathsf{RLD})_{\mathrm{in}} f, G' \sqsubseteq (\mathsf{RLD})_{\mathrm{in}} g} \right\} =$$

$$\bigsqcup^{\mathsf{RLD}} \left\{ \frac{G' \circ F'}{F' \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} f, G' \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} g} \right\} = (\mathsf{RLD})_{\mathrm{in}} g \circ (\mathsf{RLD})_{\mathrm{in}} f.$$

(*) $F' \in \operatorname{atoms}(\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} F)$ and $\operatorname{dom} F \times^{\mathsf{FCD}} \operatorname{im} F \sqsubseteq f$ implies $\operatorname{dom} F' \times^{\mathsf{FCD}} \operatorname{im} F' \sqsubseteq f$; thus $\operatorname{dom} F' \times^{\mathsf{RLD}} \operatorname{im} F' \sqsubseteq (\mathsf{RLD})_{\mathrm{in}} f$ and thus $F' \sqsubseteq (\mathsf{RLD})_{\mathrm{in}} f$. Likewise for G and G'.

Thus $(\mathsf{RLD})_{\mathrm{in}} g \circ (\mathsf{RLD})_{\mathrm{in}} f \sqsupseteq \bigsqcup^{\mathsf{RLD}} \left\{ \frac{\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} G}{F \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} f, G \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} g, \operatorname{im} F \not\asymp \operatorname{dom} G} \right\}$.

But $(\mathsf{RLD})_{\mathrm{in}} g \circ (\mathsf{RLD})_{\mathrm{in}} f \sqsubseteq \bigsqcup^{\mathsf{RLD}} \left\{ \frac{(\operatorname{dom} G \times^{\mathsf{RLD}} \operatorname{im} G) \circ (\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} F)}{F \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} f, G \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} g} \right\} = \bigsqcup^{\mathsf{RLD}} \left\{ \frac{\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} G}{F \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} f, G \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} g, \operatorname{im} F \not\asymp \operatorname{dom} G} \right\}$.

Thus

$$(\mathsf{RLD})_{\mathrm{in}} g \circ (\mathsf{RLD})_{\mathrm{in}} f = \bigsqcup^{\mathsf{RLD}} \left\{ \frac{\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} G}{F \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} f, G \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}} g, \operatorname{im} F \not\asymp \operatorname{dom} G} \right\} =$$

$$\bigsqcup^{\mathsf{RLD}} \left\{ \frac{\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} G}{\begin{array}{l} F \in \operatorname{atoms}^{\mathsf{RLD}(\operatorname{Src} f, \operatorname{Dst} f)}, G \in \operatorname{atoms}^{\mathsf{RLD}(\operatorname{Dst} f, \operatorname{Dst} g)}, \\ \operatorname{dom} F \times^{\mathsf{FCD}} \operatorname{im} F \sqsubseteq f, \operatorname{dom} G \times^{\mathsf{FCD}} \operatorname{im} G \sqsubseteq g, \operatorname{im} F \not\asymp \operatorname{dom} G \end{array}} \right\}.$$

But

$$(\mathsf{RLD})_{\mathrm{in}} (g \circ f) = \bigsqcup \left\{ \frac{a \times^{\mathsf{RLD}} c}{a \times^{\mathsf{FCD}} c \in \operatorname{atoms}(g \circ f)} \right\} = (\text{proposition } 907) =$$

$$\bigsqcup \left\{ \frac{a \times^{\mathsf{RLD}} c}{a \in \mathscr{F}(\operatorname{Src} f), c \in \mathscr{F}(\operatorname{Dst} g), \exists b \in \mathscr{F}(\operatorname{Dst} f) : (a \times^{\mathsf{FCD}} b \in \operatorname{atoms} f \wedge b \times^{\mathsf{FCD}} c \in \operatorname{atoms} g)} \right\} =$$

$$\bigsqcup \left\{ \frac{a \times^{\mathsf{RLD}} c}{a \in \mathscr{F}(\operatorname{Src} f), c \in \mathscr{F}(\operatorname{Dst} g), \exists b_0, b_1 \in \mathscr{F}(\operatorname{Dst} f) : (a \times^{\mathsf{FCD}} b \in \operatorname{atoms} f \wedge b \times^{\mathsf{FCD}} c \in \operatorname{atoms} g \wedge b_0 \not\asymp b_1)} \right\}.$$

Now it becomes obvious that $(\mathsf{RLD})_{\mathrm{in}} g \circ (\mathsf{RLD})_{\mathrm{in}} f = (\mathsf{RLD})_{\mathrm{in}} (g \circ f)$. □

9.5. Complete funcoids and reloids

For the proof below assume

$$\theta = \left(\bigsqcup_{x \in \operatorname{Src} f} (\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{RLD}} \langle f \rangle^* @\{x\}) \mapsto \bigsqcup_{x \in \operatorname{Src} f} (\uparrow^{\operatorname{Src} f} \{x\} \times^{\mathsf{FCD}} \langle f \rangle^* @\{x\}) \right)$$

(where f ranges the set of complete funcoids).

LEMMA 1119. θ is a bijection from complete reloids into complete funcoids.

PROOF. Theorems 928 and 1039. □

LEMMA 1120. $(\mathsf{FCD}) g = \theta g$ for every complete reloid g.