It remains to prove $(\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})f = f$ for a funcoidal reloid f. ($(\mathsf{FCD})(\mathsf{RLD})_{\mathrm{in}}g = g$ for every funcoid g is already proved above.)

$$(\mathsf{RLD})_{\mathrm{in}}(\mathsf{FCD})f = \bigsqcup\left\{\frac{x \times^{\mathsf{RLD}} y}{x \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Src} f)}, y \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Dst} f)}, x \times^{\mathsf{RLD}} y \not\asymp f}\right\} = \bigsqcup\left\{\frac{p \in \operatorname{atoms}(x \times^{\mathsf{RLD}} y)}{x \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Src} f)}, y \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Dst} f)}, x \times^{\mathsf{RLD}} y \not\asymp f}\right\} = \bigsqcup\left\{\frac{p \in \operatorname{atoms}(x \times^{\mathsf{RLD}} y)}{x \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Src} f)}, y \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Dst} f)}, x \times^{\mathsf{RLD}} y \sqsubseteq f}\right\} = \bigsqcup \operatorname{atoms} f = f.$$

□

Corollary 1117. Funcoidal reloids are convex.

Proof. Every $(\mathsf{RLD})_{\mathrm{in}}f$ is obviously convex. □

Theorem 1118. $(\mathsf{RLD})_{\mathrm{in}}(g \circ f) = (\mathsf{RLD})_{\mathrm{in}}g \circ (\mathsf{RLD})_{\mathrm{in}}f$ for every composable funcoids f and g.

Proof.

$$(\mathsf{RLD})_{\mathrm{in}}g \circ (\mathsf{RLD})_{\mathrm{in}}f = (\text{corollary } 1008) = \bigsqcup^{\mathsf{RLD}}\left\{\frac{G \circ F}{F \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}}f, G \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}}g}\right\}$$

Let F be an atom of the poset $\mathsf{RLD}(\operatorname{Src} f, \operatorname{Dst} f)$.

$$F \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}}f \Rightarrow \operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} F \not\asymp (\mathsf{RLD})_{\mathrm{in}}f \Rightarrow (\text{because } (\mathsf{RLD})_{\mathrm{in}}f \text{ is a funcoidal reloid}) \Rightarrow \operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} F \sqsubseteq (\mathsf{RLD})_{\mathrm{in}}f$$

but $\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} F \sqsubseteq (\mathsf{RLD})_{\mathrm{in}}f \Rightarrow F \sqsubseteq (\mathsf{RLD})_{\mathrm{in}}f$ is obvious.

So

$$F \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}}f \Leftrightarrow \operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} F \sqsubseteq (\mathsf{RLD})_{\mathrm{in}}f \Rightarrow (\mathsf{FCD})(\operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} F) \sqsubseteq (\mathsf{FCD})(\mathsf{RLD})_{\mathrm{in}}f \Leftrightarrow \operatorname{dom} F \times^{\mathsf{FCD}} \operatorname{im} F \sqsubseteq f.$$

But

$$\operatorname{dom} F \times^{\mathsf{FCD}} \operatorname{im} F \sqsubseteq f \Rightarrow (\mathsf{RLD})_{\mathrm{in}}(\operatorname{dom} F \times^{\mathsf{FCD}} \operatorname{im} F) \sqsubseteq (\mathsf{RLD})_{\mathrm{in}}f \Leftrightarrow \operatorname{dom} F \times^{\mathsf{RLD}} \operatorname{im} F \sqsubseteq (\mathsf{RLD})_{\mathrm{in}}f.$$

So $F \in \operatorname{atoms}(\mathsf{RLD})_{\mathrm{in}}f \Leftrightarrow \operatorname{dom} F \times^{\mathsf{FCD}} \operatorname{im} F \sqsubseteq f$.