

EXERCISE 1103. Prove that $\text{card FCD}(A, B) = 2^{2^{\max\{A, B\}}}$ if A or B is an infinite set (provided that A and B are nonempty).

LEMMA 1104. $\uparrow^{\text{FCD}(\text{Src } g, \text{Dst } g)} \{(x, y)\} \sqsubseteq (\text{FCD})g \Leftrightarrow \uparrow^{\text{RLD}(\text{Src } g, \text{Dst } g)} \{(x, y)\} \sqsubseteq g$ for every reloid g .

PROOF.

$$\begin{aligned} \uparrow^{\text{FCD}(\text{Src } g, \text{Dst } g)} \{(x, y)\} \sqsubseteq (\text{FCD})g &\Leftrightarrow \\ \uparrow^{\text{FCD}(\text{Src } g, \text{Dst } g)} \{(x, y)\} \not\sqsubseteq (\text{FCD})g &\Leftrightarrow @\{x\} [(\text{FCD})g]^* @\{y\} \Leftrightarrow \\ \uparrow^{\text{RLD}(\text{Src } g, \text{Dst } g)} \{(x, y)\} \not\sqsubseteq g &\Leftrightarrow \uparrow^{\text{RLD}(\text{Src } g, \text{Dst } g)} \{(x, y)\} \sqsubseteq g. \end{aligned}$$

□

THEOREM 1105. $\text{Cor}(\text{FCD})g = (\text{FCD})\text{Cor } g$ for every reloid g .

PROOF.

$$\begin{aligned} \text{Cor}(\text{FCD})g &= \\ \bigsqcup \left\{ \frac{\uparrow^{\text{FCD}(\text{Src } g, \text{Dst } g)} \{(x, y)\}}{\uparrow^{\text{FCD}} \{(x, y)\} \sqsubseteq (\text{FCD})g} \right\} &= \\ \bigsqcup \left\{ \frac{\uparrow^{\text{FCD}(\text{Src } g, \text{Dst } g)} \{(x, y)\}}{\uparrow^{\text{RLD}(\text{Src } g, \text{Dst } g)} \{(x, y)\} \sqsubseteq g} \right\} &= \\ \bigsqcup \left\{ \frac{(\text{FCD}) \uparrow^{\text{RLD}(\text{Src } g, \text{Dst } g)} \{(x, y)\}}{\uparrow^{\text{RLD}(\text{Src } g, \text{Dst } g)} \{(x, y)\} \sqsubseteq g} \right\} &= \\ (\text{FCD}) \bigsqcup \left\{ \frac{\uparrow^{\text{RLD}(\text{Src } g, \text{Dst } g)} \{(x, y)\}}{\uparrow^{\text{RLD}(\text{Src } g, \text{Dst } g)} \{(x, y)\} \sqsubseteq g} \right\} &= \\ &(\text{FCD})\text{Cor } g. \end{aligned}$$

□

CONJECTURE 1106.

- 1°. $\text{Cor}(\text{RLD})_{\text{in}}g = (\text{RLD})_{\text{in}}\text{Cor } g$;
- 2°. $\text{Cor}(\text{RLD})_{\text{out}}g = (\text{RLD})_{\text{out}}\text{Cor } g$.

THEOREM 1107. For every reloid f :

- 1°. $\text{Compl}(\text{FCD})f = (\text{FCD})\text{Compl } f$;
- 2°. $\text{CoCompl}(\text{FCD})f = (\text{FCD})\text{CoCompl } f$.

PROOF. We will prove only the first, because the second is dual.

$$\begin{aligned} \text{Compl}(\text{FCD})f &= \bigsqcup_{\alpha \in \text{Src } f} ((\text{FCD})f)|_{\uparrow\{\alpha\}} = (\text{proposition 1073}) = \\ \bigsqcup_{\alpha \in \text{Src } f} (\text{FCD})(f|_{\uparrow\{\alpha\}}) &= (\text{FCD}) \bigsqcup_{\alpha \in \text{Src } f} f|_{\uparrow\{\alpha\}} = (\text{FCD})\text{Compl } f. \end{aligned}$$

□

CONJECTURE 1108.

- 1°. $\text{Compl}(\text{RLD})_{\text{in}}g = (\text{RLD})_{\text{in}}\text{Compl } g$;
- 2°. $\text{Compl}(\text{RLD})_{\text{out}}g = (\text{RLD})_{\text{out}}\text{Compl } g$.

Note that the above Galois connection between funcoids and reloids is a Galois surjection.

$$\text{PROPOSITION 1109. } (\text{RLD})_{\text{in}}g = \max \left\{ \frac{f \in \text{RLD}}{(\text{FCD})f \sqsubseteq g} \right\} = \max \left\{ \frac{f \in \text{RLD}}{(\text{FCD})f = g} \right\}.$$

PROOF. By theorem 131 and proposition 320.

□