PROOF. Recall that $\mathrm{id}^{\mathsf{RLD}}_{\mathcal{A}} = \bigsqcap\left\{\frac{\uparrow^{\mathrm{Base}(\mathcal{A})}\mathrm{id}_A}{A\in\mathrm{up}\,\mathcal{A}}\right\}$. For every $\mathcal{X},\mathcal{Y}\in\mathscr{F}(\mathrm{Base}(\mathcal{A}))$ we have

$$\begin{aligned}\mathcal{X}\left[(\mathsf{FCD})\,\mathrm{id}^{\mathsf{RLD}}_{\mathcal{A}}\right]\mathcal{Y}\Leftrightarrow\\ \mathcal{X}\times^{\mathsf{RLD}}\mathcal{Y}\not\asymp\mathrm{id}^{\mathsf{RLD}}_{\mathcal{A}}\Leftrightarrow\\ \forall A\in\mathrm{up}\,\mathcal{A}:\mathcal{X}\times^{\mathsf{RLD}}\mathcal{Y}\not\asymp\uparrow^{\mathsf{RLD}(\mathrm{Base}(\mathcal{A}),\mathrm{Base}(\mathcal{A}))}\mathrm{id}_A\Leftrightarrow\\ \forall A\in\mathrm{up}\,\mathcal{A}:\mathcal{X}\left[\uparrow^{\mathsf{FCD}(\mathrm{Base}(\mathcal{A}),\mathrm{Base}(\mathcal{A}))}\mathrm{id}_A\right]\mathcal{Y}\Leftrightarrow\\ \forall A\in\mathrm{up}\,\mathcal{A}:\mathcal{X}\sqcap\mathcal{Y}\not\asymp A\Leftrightarrow\\ \mathcal{X}\sqcap\mathcal{Y}\not\asymp\mathcal{A}\Leftrightarrow\\ \mathcal{X}\left[\mathrm{id}^{\mathsf{FCD}}_{\mathcal{A}}\right]\mathcal{Y}\end{aligned}$$

(used properties of generalized filter bases). □

COROLLARY 1068. $(\mathsf{FCD})1^{\mathsf{RLD}}_A = 1^{\mathsf{FCD}}_A$ for every set A.

PROPOSITION 1069. (FCD) is a functor from RLD to FCD.

PROOF. Preservation of composition and of identity is proved above. □

PROPOSITION 1070.

1°. $(\mathsf{FCD})f$ is a monovalued funcoid if f is a monovalued reloid.
2°. $(\mathsf{FCD})f$ is an injective funcoid if f is an injective reloid.

PROOF. We will prove only the first as the second is dual. Let f be a monovalued reloid. Then $f\circ f^{-1}\sqsubseteq 1^{\mathsf{RLD}}_{\mathrm{Dst}\,f}$; $(\mathsf{FCD})(f\circ f^{-1})\sqsubseteq 1^{\mathsf{FCD}}_{\mathrm{Dst}\,f}$; $(\mathsf{FCD})f\circ((\mathsf{FCD})f)^{-1}\sqsubseteq 1^{\mathsf{FCD}}_{\mathrm{Dst}\,f}$ that is $(\mathsf{FCD})f$ is a monovalued funcoid. □

PROPOSITION 1071. $(\mathsf{FCD})(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B}) = \mathcal{A}\times^{\mathsf{FCD}}\mathcal{B}$ for every filters $\mathcal{A}$, $\mathcal{B}$.

PROOF. $\mathcal{X}\left[(\mathsf{FCD})(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B})\right]\mathcal{Y}\Leftrightarrow\forall F\in\mathrm{up}(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B}):\mathcal{X}\left[\uparrow^{\mathsf{FCD}}F\right]\mathcal{Y}$ (for every $\mathcal{X}\in\mathscr{F}(\mathrm{Base}(\mathcal{A}))$, $\mathcal{Y}\in\mathscr{F}(\mathrm{Base}(\mathcal{B}))$.

Evidently

$$\forall F\in\mathrm{up}(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B}):\mathcal{X}\left[\uparrow^{\mathsf{FCD}}F\right]\mathcal{Y}\Rightarrow\forall A\in\mathrm{up}\,\mathcal{A},B\in\mathrm{up}\,\mathcal{B}:\mathcal{X}\left[A\times B\right]\mathcal{Y}.$$

Let $\forall A\in\mathrm{up}\,\mathcal{A},B\in\mathrm{up}\,\mathcal{B}:\mathcal{X}\left[A\times B\right]\mathcal{Y}$. Then if $F\in\mathrm{up}(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B})$, there are $A\in\mathrm{up}\,\mathcal{A}$, $B\in\mathrm{up}\,\mathcal{B}$ such that $F\sqsupseteq A\times B$. So $\mathcal{X}\left[\uparrow^{\mathsf{FCD}}F\right]\mathcal{Y}$. We have proved

$$\forall F\in\mathrm{up}(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B}):\mathcal{X}\left[\uparrow^{\mathsf{FCD}}F\right]\mathcal{Y}\Leftrightarrow\forall A\in\mathrm{up}\,\mathcal{A},B\in\mathrm{up}\,\mathcal{B}:\mathcal{X}\left[A\times B\right]\mathcal{Y}.$$

Further

$$\begin{aligned}&\forall A\in\mathrm{up}\,\mathcal{A},B\in\mathrm{up}\,\mathcal{B}:\mathcal{X}\left[A\times B\right]\mathcal{Y}\Leftrightarrow\\ &\qquad\forall A\in\mathrm{up}\,\mathcal{A},B\in\mathrm{up}\,\mathcal{B}:(\mathcal{X}\not\asymp A\wedge\mathcal{Y}\not\asymp B)\Leftrightarrow\\ &\qquad\qquad\mathcal{X}\not\asymp\mathcal{A}\wedge\mathcal{Y}\not\asymp\mathcal{B}\Leftrightarrow\mathcal{X}\left[\mathcal{A}\times^{\mathsf{FCD}}\mathcal{B}\right]\mathcal{Y}.\end{aligned}$$

Thus $\mathcal{X}\left[(\mathsf{FCD})(\mathcal{A}\times^{\mathsf{RLD}}\mathcal{B})\right]\mathcal{Y}\Leftrightarrow\mathcal{X}\left[\mathcal{A}\times^{\mathsf{FCD}}\mathcal{B}\right]\mathcal{Y}$. □

PROPOSITION 1072. $\mathrm{dom}(\mathsf{FCD})f = \mathrm{dom}\,f$ and $\mathrm{im}(\mathsf{FCD})f = \mathrm{im}\,f$ for every reloid f.