Obviously

$$\bigsqcap^{\mathsf{RLD}}\left\{\frac{G\circ F}{F\in\operatorname{up} f, G\in\operatorname{up} g}\right\}=\bigsqcap^{\mathsf{RLD}}\operatorname{up}\bigsqcap^{\mathsf{RLD}}\left\{\frac{G\circ F}{F\in\operatorname{up} f, G\in\operatorname{up} g}\right\};$$

from this by lemma 1064 (taking into account that

$$\left\{\frac{G\circ F}{F\in\operatorname{up} f, G\in\operatorname{up} g}\right\}$$

and

$$\operatorname{up}\bigsqcap^{\mathsf{RLD}}\left\{\frac{G\circ F}{F\in\operatorname{up} f, G\in\operatorname{up} g}\right\}$$

are filter bases)

$$\bigsqcap^{\mathsf{RLD}}_{H\in\operatorname{up}\bigsqcap^{\mathsf{RLD}}\left\{\frac{G\circ F}{F\in\operatorname{up} f, G\in\operatorname{up} g}\right\}}\langle H\rangle^* X=\bigsqcap^{\mathscr{F}}\left\{\frac{\langle G\circ F\rangle^* X}{F\in\operatorname{up} f, G\in\operatorname{up} g}\right\}.$$

On the other side

$$\langle((\mathsf{FCD})g)\circ((\mathsf{FCD})f)\rangle^* X=\langle(\mathsf{FCD})g\rangle\langle(\mathsf{FCD})f\rangle^* X=$$
$$\langle(\mathsf{FCD})g\rangle\bigsqcap^{\mathscr{F}}_{F\in\operatorname{up} f}\langle F\rangle^* X=\bigsqcap_{G\in\operatorname{up} g}\langle\uparrow^{\mathsf{FCD}} G\rangle\bigsqcap^{\mathsf{RLD}}_{F\in\operatorname{up} f}\langle F\rangle^* X.$$

Let's prove that $\left\{\frac{\langle F\rangle^* X}{F\in\operatorname{up} f}\right\}$ is a filter base. If $A,B\in\left\{\frac{\langle F\rangle^* X}{F\in\operatorname{up} f}\right\}$ then $A=\langle F_1\rangle^* X$, $B=\langle F_2\rangle^* X$ where $F_1,F_2\in\operatorname{up} f$. $A\sqcap B\sqsupseteq\langle F_1\sqcap F_2\rangle^* X\in\left\{\frac{\langle F\rangle^* X}{F\in\operatorname{up} f}\right\}$. So $\left\{\frac{\langle F\rangle^* X}{F\in\operatorname{up} f}\right\}$ is really a filter base.

By theorem 836 we have

$$\langle\uparrow^{\mathsf{FCD}} G\rangle\bigsqcap^{\mathscr{F}}_{F\in\operatorname{up} f}\langle F\rangle^* X=\bigsqcap^{\mathscr{F}}_{F\in\operatorname{up} f}\langle G\rangle^*\langle F\rangle^* X.$$

So continuing the above equalities,

$$\langle((\mathsf{FCD})g)\circ((\mathsf{FCD})f)\rangle^* X=$$
$$\bigsqcap^{\mathscr{F}}_{G\in\operatorname{up} g}\bigsqcap^{\mathscr{F}}_{F\in\operatorname{up} f}\langle G\rangle^*\langle F\rangle^* X=$$
$$\bigsqcap^{\mathscr{F}}\left\{\frac{\langle G\rangle^*\langle F\rangle^* X}{F\in\operatorname{up} f, G\in\operatorname{up} g}\right\}=$$
$$\bigsqcap^{\mathscr{F}}\left\{\frac{\langle G\circ F\rangle^* X}{F\in\operatorname{up} f, G\in\operatorname{up} g}\right\}.$$

Combining these equalities we get $\langle(\mathsf{FCD})(g\circ f)\rangle^* X=\langle((\mathsf{FCD})g)\circ((\mathsf{FCD})f)\rangle^* X$ for every typed set $X\in\mathscr{T}(\operatorname{Src} f)$. □

Proposition 1067. $(\mathsf{FCD})\operatorname{id}^{\mathsf{RLD}}_{\mathcal{A}}=\operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}}$ for every filter $\mathcal{A}$.