PROOF.

$$\begin{aligned}\mathcal{X} \times^{\mathsf{RLD}} \mathcal{Y} \not\asymp f &\Leftrightarrow \\ \forall F \in \operatorname{up} f, P \in \operatorname{up}(\mathcal{X} \times^{\mathsf{RLD}} \mathcal{Y}) : P \not\asymp F &\Leftrightarrow \\ \forall F \in \operatorname{up} f, X \in \operatorname{up} \mathcal{X}, Y \in \operatorname{up} \mathcal{Y} : X \times Y \not\asymp F &\Leftrightarrow \\ \forall F \in \operatorname{up} f, X \in \operatorname{up} \mathcal{X}, Y \in \operatorname{up} \mathcal{Y} : X \left[\uparrow^{\mathsf{FCD}} F\right] Y &\Leftrightarrow \\ \forall F \in \operatorname{up} f : \mathcal{X} \left[\uparrow^{\mathsf{FCD}} F\right] \mathcal{Y} &\Leftrightarrow \\ \mathcal{X} \left[(\mathsf{FCD}) f\right] \mathcal{Y}. &\end{aligned}$$

□

THEOREM 1063. $(\mathsf{FCD})f = \bigsqcap^{\mathsf{FCD}} \operatorname{up} f$ for every reloid f.

PROOF. Let a be an ultrafilter on $\operatorname{Src} f$.

$\langle (\mathsf{FCD})f \rangle a = \bigsqcap \left\{ \frac{\langle \uparrow^{\mathsf{FCD}} F \rangle a}{F \in \operatorname{up} f} \right\}$ by the definition of (FCD).

$\left\langle \bigsqcap^{\mathsf{FCD}} \operatorname{up} f \right\rangle a = \bigsqcap \left\{ \frac{\langle \uparrow^{\mathsf{FCD}} F \rangle a}{F \in \operatorname{up} f} \right\}$ by theorem 875.

So $\langle (\mathsf{FCD})f \rangle a = \left\langle \bigsqcap^{\mathsf{FCD}} \operatorname{up} f \right\rangle a$ for every ultrafilter a. □

LEMMA 1064. For every two filter bases S and T of morphisms $\mathbf{Rel}(U, V)$ and every typed set $A \in \mathscr{T} U$

$$\bigsqcap^{\mathsf{RLD}} S = \bigsqcap^{\mathsf{RLD}} T \Rightarrow \bigsqcap^{\mathscr{F}}_{F \in S} \langle F \rangle^* A = \bigsqcap^{\mathscr{F}}_{G \in T} \langle G \rangle^* A.$$

PROOF. Let $\bigsqcap^{\mathsf{RLD}} S = \bigsqcap^{\mathsf{RLD}} T$.

First let prove that $\left\{ \frac{\langle F \rangle^* A}{F \in S} \right\}$ is a filter base. Let $X, Y \in \left\{ \frac{\langle F \rangle^* A}{F \in S} \right\}$. Then $X = \langle F_X \rangle^* A$ and $Y = \langle F_Y \rangle^* A$ for some $F_X, F_Y \in S$. Because S is a filter base, we have $S \ni F_Z \sqsubseteq F_X \sqcap F_Y$. So $\langle F_Z \rangle^* A \sqsubseteq X \sqcap Y$ and $\langle F_Z \rangle^* A \in \left\{ \frac{\langle F \rangle^* A}{F \in S} \right\}$. So $\left\{ \frac{\langle F \rangle^* A}{F \in S} \right\}$ is a filter base.

Suppose $X \in \operatorname{up} \bigsqcap^{\mathscr{F}}_{F \in S} \langle F \rangle^* A$. Then there exists $X' \in \left\{ \frac{\langle F \rangle^* A}{F \in S} \right\}$ where $X \sqsupseteq X'$ because $\left\{ \frac{\langle F \rangle^* A}{F \in S} \right\}$ is a filter base. That is $X' = \langle F \rangle^* A$ for some $F \in S$. There exists $G \in T$ such that $G \sqsubseteq F$ because T is a filter base. Let $Y' = \langle G \rangle^* A$. We have $Y' \sqsubseteq X' \sqsubseteq X$; $Y' \in \left\{ \frac{\langle G \rangle^* A}{G \in T} \right\}$; $Y' \in \operatorname{up} \bigsqcap^{\mathscr{F}}_{G \in T} \langle G \rangle^* A$; $X \in \operatorname{up} \bigsqcap^{\mathscr{F}}_{G \in T} \langle G \rangle^* A$. The reverse is symmetric. □

LEMMA 1065. $\left\{ \frac{G \circ F}{F \in \operatorname{up} f, G \in \operatorname{up} g} \right\}$ is a filter base for every reloids f and g.

PROOF. Let denote $D = \left\{ \frac{G \circ F}{F \in \operatorname{up} f, G \in \operatorname{up} g} \right\}$. Let $A \in D \wedge B \in D$. Then $A = G_A \circ F_A \wedge B = G_B \circ F_B$ for some $F_A, F_B \in \operatorname{up} f$, $G_A, G_B \in \operatorname{up} g$. So $A \sqcap B \sqsupseteq (G_A \sqcap G_B) \circ (F_A \sqcap F_B) \in D$ because $F_A \sqcap F_B \in \operatorname{up} f$ and $G_A \sqcap G_B \in \operatorname{up} g$. □

THEOREM 1066. $(\mathsf{FCD})(g \circ f) = ((\mathsf{FCD})g) \circ ((\mathsf{FCD})f)$ for every composable reloids f and g.

PROOF.

$$\langle (\mathsf{FCD})(g \circ f) \rangle^* X = \bigsqcap^{\mathscr{F}}_{H \in \operatorname{up}(g \circ f)} \langle H \rangle^* X = \bigsqcap^{\mathscr{F}}_{H \in \operatorname{up} \bigsqcap^{\mathsf{RLD}} \left\{ \frac{G \circ F}{F \in \operatorname{up} f, G \in \operatorname{up} g} \right\}} \langle H \rangle^* X.$$