PROOF.

$$\operatorname{Compl} \bigsqcup R = \bigsqcup \left\{ \frac{(\bigsqcup R)|_{\uparrow^A \{\alpha\}}}{\alpha \in A} \right\} = \text{(theorem 607)} \bigsqcup \left\{ \frac{\bigsqcup \left\{ \frac{f|_{\uparrow\{\alpha\}}}{\alpha \in A} \right\}}{f \in R} \right\} = \bigsqcup \langle \operatorname{Compl} \rangle^* R.$$

□

LEMMA 1053. Completion of a co-complete reloid is principal.

PROOF. Let f be a co-complete reloid. Then there is a function $F : \operatorname{Dst} f \to \mathscr{F}(\operatorname{Src} f)$ such that

$$f = \bigsqcup \left\{ \frac{F(\alpha) \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\}}{\alpha \in \operatorname{Dst} f} \right\}.$$

So

$$\begin{aligned} &\operatorname{Compl} f = \\ &\bigsqcup \left\{ \frac{\left(\bigsqcup \left\{ \frac{F(\alpha) \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\}}{\alpha \in \operatorname{Dst} f} \right\} \right)|_{\uparrow\{\beta\}}}{\beta \in \operatorname{Src} f} \right\} = \\ &\bigsqcup \left\{ \frac{\left(\bigsqcup \left\{ \frac{F(\alpha) \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\}}{\alpha \in \operatorname{Dst} f} \right\} \right) \sqcap (\uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} \top^{\mathscr{F}(\operatorname{Dst} f)})}{\beta \in \operatorname{Src} f} \right\} = (*) \\ &\bigsqcup \left\{ \frac{\bigsqcup \left\{ \frac{(F(\alpha) \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\}) \sqcap (\uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} \top^{\mathscr{F}(\operatorname{Dst} f)})}{\alpha \in \operatorname{Dst} f} \right\}}{\beta \in \operatorname{Src} f} \right\} = \\ &\bigsqcup \left\{ \frac{\bigsqcup \left\{ \frac{\uparrow^{\operatorname{Src} f} \{\beta\} \times^{\mathsf{RLD}} \uparrow^{\operatorname{Dst} f} \{\alpha\}}{\alpha \in \operatorname{Dst} f} \right\}}{\beta \in \operatorname{Src} f, \uparrow^{\operatorname{Src} f} \{\beta\} \sqsubseteq F(\alpha)} \right\} \end{aligned}$$

* theorem 607.

Thus $\operatorname{Compl} f$ is principal. □

THEOREM 1054. $\operatorname{Compl} \operatorname{CoCompl} f = \operatorname{CoCompl} \operatorname{Compl} f = \operatorname{Cor} f$ for every reloid f.

PROOF. We will prove only $\operatorname{Compl} \operatorname{CoCompl} f = \operatorname{Cor} f$. The rest follows from symmetry.

From the lemma $\operatorname{Compl} \operatorname{CoCompl} f$ is principal. It is obvious $\operatorname{Compl} \operatorname{CoCompl} f \sqsubseteq f$. So to finish the proof we need to show only that for every principal reloid $F \sqsubseteq f$ we have $F \sqsubseteq \operatorname{Compl} \operatorname{CoCompl} f$.

Really, obviously $F \sqsubseteq \operatorname{CoCompl} f$ and thus $F = \operatorname{Compl} F \sqsubseteq \operatorname{Compl} \operatorname{CoCompl} f$. □

CONJECTURE 1055. If f is a complete reloid, then it is metacomplete.

CONJECTURE 1056. If f is a metacomplete reloid, then it is complete.

CONJECTURE 1057. $\operatorname{Compl} f = f \setminus^* (\Omega^{\operatorname{Src} f} \times^{\mathsf{RLD}} \top^{\mathscr{F}(\operatorname{Dst} f)})$ for every reloid f.

By analogy with similar properties of funcoids described above: