Denote $f = \bigsqcup\left\{\frac{\uparrow^A\{\alpha\}\times^{\mathsf{RLD}}G(\alpha)}{\alpha\in A}\right\}$. Then

$$\operatorname{im}(f|_{\uparrow\{\alpha'\}}) = \operatorname{im}(f\sqcap(\uparrow^A\{\alpha'\}\times\top^{\mathscr{F}(B)})) = (\text{because } \uparrow^A\{\alpha'\}\times\top^{\mathscr{F}(B)} \text{ is principal}) =$$

$$\operatorname{im}\bigsqcup\left\{\frac{(\uparrow^A\{\alpha\}\times^{\mathsf{RLD}}G(\alpha))\sqcap(\uparrow^A\{\alpha'\}\times\top^{\mathscr{F}(B)})}{\alpha\in\operatorname{Src}f}\right\} = \operatorname{im}(\uparrow^A\{\alpha'\}\times^{\mathsf{RLD}}G(\alpha')) = G(\alpha').$$

□

Corollary 1040. $G\mapsto\bigsqcup\left\{\frac{G(\alpha)\times^{\mathsf{RLD}}\uparrow^A\{\alpha\}}{\alpha\in A}\right\}$ is an order isomorphism from the set of functions $G\in\mathscr{F}(B)^A$ to the set $\mathsf{CoComplRLD}(A,B)$.

The inverse isomorphism is described by the formula $G(\alpha)=\operatorname{im}(f^{-1}|_{\uparrow\{\alpha\}})$ where f is a co-complete reloid.

Corollary 1041. $\mathsf{ComplRLD}(A,B)$ and $\mathsf{ComplFCD}(A,B)$ are a co-frames.

Obvious 1042. Complete and co-complete reloids are convex.

Obvious 1043. Principal reloids are complete and co-complete.

Obvious 1044. Join (on the lattice of reloids) of complete reloids is complete.

Theorem 1045. A reloid which is both complete and co-complete is principal.

Proof. Let f be a complete and co-complete reloid. We have

$$f=\bigsqcup\left\{\frac{\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}G(\alpha)}{\alpha\in\operatorname{Src}f}\right\}\quad\text{and}\quad f=\bigsqcup\left\{\frac{H(\beta)\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst}f}\{\beta\}}{\beta\in\operatorname{Dst}f}\right\}$$

for some functions $G:\operatorname{Src}f\to\mathscr{F}(\operatorname{Dst}f)$ and $H:\operatorname{Dst}f\to\mathscr{F}(\operatorname{Src}f)$. For every $\alpha\in\operatorname{Src}f$ we have

$$\begin{aligned}
&G(\alpha)=\\
&\operatorname{im}f|_{\uparrow\{\alpha\}}=\\
&\operatorname{im}(f\sqcap(\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}\top^{\mathscr{F}(\operatorname{Dst}f)}))=\quad(*)\\
&\operatorname{im}\bigsqcup\left\{\frac{(H(\beta)\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst}f}\{\beta\})\sqcap(\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}\top^{\mathscr{F}(\operatorname{Dst}f)})}{\beta\in\operatorname{Dst}f}\right\}=\\
&\operatorname{im}\bigsqcup\left\{\frac{(H(\beta)\sqcap\uparrow^{\operatorname{Src}f}\{\alpha\})\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst}f}\{\beta\}}{\beta\in\operatorname{Dst}f}\right\}=\\
&\operatorname{im}\bigsqcup\left\{\frac{\left(\begin{cases}\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst}f}\{\beta\} & \text{if } H(\beta)\not\asymp\uparrow^{\operatorname{Src}f}\{\alpha\}\\ \bot^{\mathsf{RLD}(\operatorname{Src}f,\operatorname{Dst}f)} & \text{if } H(\beta)\asymp\uparrow^{\operatorname{Src}f}\{\alpha\}\end{cases}\right)}{\beta\in\operatorname{Dst}f}\right\}=\\
&\operatorname{im}\bigsqcup\left\{\frac{\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}\uparrow^{\operatorname{Dst}f}\{\beta\}}{\beta\in\operatorname{Dst}f,H(\beta)\not\asymp\uparrow^{\operatorname{Src}f}\{\alpha\}}\right\}=\\
&\operatorname{im}\bigsqcup\left\{\frac{\uparrow^{\mathsf{RLD}(\operatorname{Src}f,\operatorname{Dst}f)}\{(\alpha,\beta)\}}{\beta\in\operatorname{Dst}f,H(\beta)\not\asymp\uparrow^{\operatorname{Src}f}\{\alpha\}}\right\}=\\
&\bigsqcup\left\{\frac{\uparrow^{\operatorname{Dst}f}\{\beta\}}{\beta\in\operatorname{Dst}f,H(\beta)\not\asymp\uparrow^{\operatorname{Src}f}\{\alpha\}}\right\}
\end{aligned}$$

* theorem 607 was used.

Thus $G(\alpha)$ is a principal filter that is $G(\alpha)=\uparrow^{\operatorname{Dst}f}g(\alpha)$ for some $g:\operatorname{Src}f\to\operatorname{Dst}f$; $\uparrow^{\operatorname{Src}f}\{\alpha\}\times^{\mathsf{RLD}}G(\alpha)=\uparrow^{\mathsf{RLD}(\operatorname{Src}f,\operatorname{Dst}f)}(\{\alpha\}\times g(\alpha))$; f is principal as a join of principal reloids. □