- The morphisms from a filter $\mathcal{A}$ to a filter $\mathcal{B}$ are triples $(\mathcal{A}, \mathcal{B}, f)$ where $f \in \mathsf{RLD}(\operatorname{Base}(\mathcal{A}), \operatorname{Base}(\mathcal{B}))$ and $\operatorname{dom} f \sqsubseteq \mathcal{A}$, $\operatorname{im} f \sqsubseteq \mathcal{B}$.
- The composition is defined by the formula $(\mathcal{B}, \mathcal{C}, g) \circ (\mathcal{A}, \mathcal{B}, f) = (\mathcal{A}, \mathcal{C}, g \circ f)$.
- Identity morphism for a filter $\mathcal{A}$ is $\operatorname{id}^{\mathsf{RLD}}_{\mathcal{A}}$.

To prove that it is really a category is trivial.

Proposition 1031. $\uparrow^{\mathsf{RLD}}$ is a functor from **Rel** to RLD.

Proof. $\uparrow^{\mathsf{RLD}} (g \circ f) = \uparrow^{\mathsf{RLD}} g \circ \uparrow^{\mathsf{RLD}} f$ was proved above. $\uparrow^{\mathsf{RLD}} 1^{\mathbf{Rel}}_A = 1^{\mathsf{RLD}}_A$ is by definition. □

8.6. Monovalued and injective reloids

Following the idea of definition of monovalued morphism let's call *monovalued* such a reloid f that $f \circ f^{-1} \sqsubseteq \operatorname{id}^{\mathsf{RLD}}_{\operatorname{im} f}$.

Similarly, I will call a reloid *injective* when $f^{-1} \circ f \sqsubseteq \operatorname{id}^{\mathsf{RLD}}_{\operatorname{dom} f}$.

Obvious 1032. A reloid f is

- monovalued iff $f \circ f^{-1} \sqsubseteq 1^{\mathsf{RLD}}_{\operatorname{Dst} f}$;
- injective iff $f^{-1} \circ f \sqsubseteq 1^{\mathsf{RLD}}_{\operatorname{Src} f}$.

In other words, a reloid is monovalued (injective) when it is a monovalued (injective) morphism of the category of reloids.

Monovaluedness is dual of injectivity.

Obvious 1033.

1°. A morphism $(\mathcal{A}, \mathcal{B}, f)$ of the category of reloid triples is monovalued iff the reloid f is monovalued.

2°. A morphism $(\mathcal{A}, \mathcal{B}, f)$ of the category of reloid triples is injective iff the reloid f is injective.

Theorem 1034.

1°. A reloid f is a monovalued iff there exists a **Set**-morphism (monovalued **Rel**-morphism) $F \in \operatorname{up} f$.

2°. A reloid f is a injective iff there exists an injective **Rel**-morphism $F \in \operatorname{up} f$.

3°. A reloid f is a both monovalued and injective iff there exists an injection (a monovalued and injective **Rel**-morphism = injective **Set**-morphism) $F \in \operatorname{up} f$.

Proof. The reverse implications are obvious. Let's prove the direct implications:

1°. Let f be a monovalued reloid. Then $f \circ f^{-1} \sqsubseteq 1^{\mathsf{RLD}}_{\operatorname{Dst} f}$, that is

$$\bigsqcap^{\mathsf{RLD}} \left\{ \frac{F \circ F^{-1}}{F \in \operatorname{up} f} \right\} \sqsubseteq 1^{\mathsf{RLD}}_{\operatorname{Dst} f}.$$

It's simple to show that $\left\{ \frac{F \circ F^{-1}}{F \in \operatorname{up} f} \right\}$ is a filter base. Consequently there exists $F \in \operatorname{up} f$ such that $F \circ F^{-1} \sqsubseteq 1^{\mathsf{RLD}}_{\operatorname{Dst} f}$ that is F is monovalued.

2°. Similar.

3°. Let f be a both monovalued and injective reloid. Then by proved above there exist $F, G \in \operatorname{up} f$ such that F is monovalued and G is injective. Thus $F \sqcap G \in \operatorname{up} f$ is both monovalued and injective.

□