PROOF. First,

$$\begin{aligned}
&\operatorname{im}(f|_{\uparrow\{\alpha\}}) = \\
&\bigsqcap^{\mathsf{RLD}} \langle \operatorname{im}\rangle^* \operatorname{up}(f|_{\uparrow\{\alpha\}}) = \\
&\bigsqcap^{\mathsf{RLD}} \langle \operatorname{im}\rangle^* \operatorname{up}(f \sqcap (\uparrow^{\operatorname{Src} f} \{\alpha\} \times \top^{\mathscr{F}(\operatorname{Dst} f)})) = \\
&\bigsqcap^{\mathsf{RLD}} \left\{ \frac{\operatorname{im}(F \cap (\{\alpha\} \times \top^{\mathscr{T}(\operatorname{Dst} f)}))}{F \in \operatorname{up} f} \right\} = \\
&\bigsqcap^{\mathsf{RLD}} \left\{ \frac{\operatorname{im}(F|_{\uparrow\{\alpha\}})}{F \in \operatorname{up} f} \right\}.
\end{aligned}$$

Taking this into account we have:

$$\begin{aligned}
&\uparrow^{\operatorname{Src} f} \{\alpha\} \times^{\mathsf{RLD}} \operatorname{im}(f|_{\uparrow\{\alpha\}}) = \\
&\bigsqcap^{\mathsf{RLD}} \left\{ \frac{\uparrow^{\operatorname{Src} f} \{\alpha\} \times K}{K \in \operatorname{im}(f|_{\uparrow\{\alpha\}})} \right\} = \\
&\bigsqcap^{\mathsf{RLD}} \left\{ \frac{\uparrow^{\operatorname{Src} f} \{\alpha\} \times \operatorname{im}(F|_{\uparrow\{\alpha\}})}{F \in \operatorname{up} f} \right\} = \\
&\bigsqcap^{\mathsf{RLD}} \left\{ \frac{F|_{\uparrow\{\alpha\}}}{F \in \operatorname{up} f} \right\} = \\
&\bigsqcap^{\mathsf{RLD}} \left\{ \frac{F \sqcap (\uparrow^{\operatorname{Src} f} \{\alpha\} \times \top^{\mathscr{T}(\operatorname{Dst} f)})}{F \in \operatorname{up} f} \right\} = \\
&\bigsqcap^{\mathsf{RLD}} \left\{ \frac{F}{F \in \operatorname{up} f} \right\} \sqcap \uparrow^{\mathsf{RLD}} (\uparrow^{\operatorname{Src} f} \{\alpha\} \times \top^{\mathscr{T}(\operatorname{Dst} f)}) = \\
&f \sqcap \uparrow^{\mathsf{RLD}} (\uparrow^{\operatorname{Src} f} \{\alpha\} \times \top^{\mathscr{T}(\operatorname{Dst} f)}) = \\
&f|_{\uparrow\{\alpha\}}.
\end{aligned}$$

□

LEMMA 1029. $\lambda \mathcal{B} \in \mathscr{F}(B) : \top^{\mathscr{F}} \times^{\mathsf{RLD}} \mathcal{B}$ is an upper adjoint of $\lambda f \in \mathsf{RLD}(A, B) : \operatorname{im} f$ (for every sets A, B).

PROOF. We need to prove $\operatorname{im} f \sqsubseteq \mathcal{B} \Leftrightarrow f \sqsubseteq \top^{\mathscr{F}} \times^{\mathsf{RLD}} \mathcal{B}$ what is obvious. □

COROLLARY 1030. Image and domain of reloids preserve joins.

PROOF. By properties of Galois connections and duality. □

8.5. Categories of reloids

I will define two categories, the *category of reloids* and the *category of reloid triples*.

The *category of reloids* is defined as follows:

- Objects are small sets.
- The set of morphisms from a set A to a set B is $\mathsf{RLD}(A, B)$.
- The composition is the composition of reloids.
- Identity morphism for a set is the identity reloid for that set.

To show it is really a category is trivial.

The *category of reloid triples* is defined as follows:

- Objects are small sets.