THEOREM 1011. $\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B} = \bigsqcup\left\{ \frac{a \times^{\mathsf{RLD}} b}{a \in \operatorname{atoms} \mathcal{A}, b \in \operatorname{atoms} \mathcal{B}} \right\}$ for every filters $\mathcal{A}$ and $\mathcal{B}$.

PROOF. Obviously $\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B} \sqsupseteq \bigsqcup\left\{ \frac{a \times^{\mathsf{RLD}} b}{a \in \operatorname{atoms} \mathcal{A}, b \in \operatorname{atoms} \mathcal{B}} \right\}$.

Reversely, let $K \in \operatorname{up} \bigsqcup\left\{ \frac{a \times^{\mathsf{RLD}} b}{a \in \operatorname{atoms} \mathcal{A}, b \in \operatorname{atoms} \mathcal{B}} \right\}$. Then $K \in \operatorname{up}(a \times^{\mathsf{RLD}} b)$ for every $a \in \operatorname{atoms} \mathcal{A}$, $b \in \operatorname{atoms} \mathcal{B}$. $K \sqsupseteq X_a \times Y_b$ for some $X_a \in \operatorname{up} a$, $Y_b \in \operatorname{up} b$;

$$K \sqsupseteq \bigsqcup\left\{ \frac{X_a \times Y_b}{a \in \operatorname{atoms} \mathcal{A}, b \in \operatorname{atoms} \mathcal{B}} \right\} = \bigsqcup\left\{ \frac{X_a}{a \in \operatorname{atoms} \mathcal{A}} \right\} \times \bigsqcup\left\{ \frac{Y_b}{b \in \operatorname{atoms} \mathcal{B}} \right\} \sqsupseteq A \times B$$

where $A \in \operatorname{up} \mathcal{A}$, $B \in \operatorname{up} \mathcal{B}$; $K \in \operatorname{up}(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B})$. □

THEOREM 1012. If $\mathcal{A}_0, \mathcal{A}_1 \in \mathscr{F}(A)$, $\mathcal{B}_0, \mathcal{B}_1 \in \mathscr{F}(B)$ for some sets A, B then

$$(\mathcal{A}_0 \times^{\mathsf{RLD}} \mathcal{B}_0) \sqcap (\mathcal{A}_1 \times^{\mathsf{RLD}} \mathcal{B}_1) = (\mathcal{A}_0 \sqcap \mathcal{A}_1) \times^{\mathsf{RLD}} (\mathcal{B}_0 \sqcap \mathcal{B}_1).$$

PROOF.

$$\begin{aligned} &(\mathcal{A}_0 \times^{\mathsf{RLD}} \mathcal{B}_0) \sqcap (\mathcal{A}_1 \times^{\mathsf{RLD}} \mathcal{B}_1) = \\ &\bigsqcap^{\mathsf{RLD}}\left\{ \frac{P \sqcap Q}{P \in \operatorname{up}(\mathcal{A}_0 \times^{\mathsf{RLD}} \mathcal{B}_0), Q \in \operatorname{up}(\mathcal{A}_1 \times^{\mathsf{RLD}} \mathcal{B}_1)} \right\} = \\ &\bigsqcap^{\mathsf{RLD}}\left\{ \frac{(A_0 \times B_0) \sqcap (A_1 \times B_1)}{A_0 \in \operatorname{up} \mathcal{A}_0, B_0 \in \operatorname{up} \mathcal{B}_0, A_1 \in \operatorname{up} \mathcal{A}_1, B_1 \in \operatorname{up} \mathcal{B}_1} \right\} = \\ &\bigsqcap^{\mathsf{RLD}}\left\{ \frac{(A_0 \sqcap A_1) \times (B_0 \sqcap B_1)}{A_0 \in \operatorname{up} \mathcal{A}_0, B_0 \in \operatorname{up} \mathcal{B}_0, A_1 \in \operatorname{up} \mathcal{A}_1, B_1 \in \operatorname{up} \mathcal{B}_1} \right\} = \\ &\bigsqcap^{\mathsf{RLD}}\left\{ \frac{K \times L}{K \in \operatorname{up}(\mathcal{A}_0 \sqcap \mathcal{A}_1), L \in \operatorname{up}(\mathcal{B}_0 \sqcap \mathcal{B}_1)} \right\} = \\ &(\mathcal{A}_0 \sqcap \mathcal{A}_1) \times^{\mathsf{RLD}} (\mathcal{B}_0 \sqcap \mathcal{B}_1). \end{aligned}$$

□

THEOREM 1013. If $S \in \mathscr{P}(\mathscr{F}(A) \times \mathscr{F}(B))$ for some sets A, B then

$$\bigsqcap\left\{ \frac{\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}}{(\mathcal{A}, \mathcal{B}) \in S} \right\} = \bigsqcap \operatorname{dom} S \times^{\mathsf{RLD}} \bigsqcap \operatorname{im} S.$$

PROOF. Let $\mathcal{P} = \bigsqcap \operatorname{dom} S$, $\mathcal{Q} = \bigsqcap \operatorname{im} S$; $l = \bigsqcap\left\{ \frac{\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}}{(\mathcal{A}, \mathcal{B}) \in S} \right\}$.

$\mathcal{P} \times^{\mathsf{RLD}} \mathcal{Q} \sqsubseteq l$ is obvious.

Let $F \in \operatorname{up}(\mathcal{P} \times^{\mathsf{RLD}} \mathcal{Q})$. Then there exist $P \in \operatorname{up} \mathcal{P}$ and $Q \in \operatorname{up} \mathcal{Q}$ such that $F \sqsupseteq P \times Q$.

$P = P_1 \sqcap \dots \sqcap P_n$ where $P_i \in \operatorname{dom} S$ and $Q = Q_1 \sqcap \dots \sqcap Q_m$ where $Q_j \in \operatorname{im} S$.

$P \times Q = \bigsqcap_{i,j} (P_i \times Q_j)$.

$P_i \times Q_j \in \operatorname{up}(\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B})$ for some $(\mathcal{A}, \mathcal{B}) \in S$. $P \times Q = \bigsqcap_{i,j} (P_i \times Q_j) \in \operatorname{up} l$. So $F \in \operatorname{up} l$. □

COROLLARY 1014. $\bigsqcap \langle \mathcal{A} \times^{\mathsf{RLD}} \rangle^* T = \mathcal{A} \times^{\mathsf{RLD}} \bigsqcap T$ if $\mathcal{A}$ is a filter and T is a set of filters with common base.

PROOF. Take $S = \{\mathcal{A}\} \times T$ where T is a set of filters.

Then $\bigsqcap\left\{ \frac{\mathcal{A} \times^{\mathsf{RLD}} \mathcal{B}}{\mathcal{B} \in T} \right\} = \mathcal{A} \times^{\mathsf{RLD}} \bigsqcap T$ that is $\bigsqcap \langle \mathcal{A} \times^{\mathsf{RLD}} \rangle^* T = \mathcal{A} \times^{\mathsf{RLD}} \bigsqcap T$. □

DEFINITION 1015. I will call a reloid *convex* iff it is a join of direct products.