THEOREM 1005. For every sets A, B, C if $g, h \in \mathsf{RLD}(A, B)$ then

1°. $f \circ (g \sqcup h) = f \circ g \sqcup f \circ h$ for every $f \in \mathsf{RLD}(B, C)$;
2°. $(g \sqcup h) \circ f = g \circ f \sqcup h \circ f$ for every $f \in \mathsf{RLD}(C, A)$.

PROOF. We'll prove only the first as the second is dual.
By the infinite distributivity law for filters we have

$$f \circ g \sqcup f \circ h = \bigsqcap^{\mathsf{RLD}} \left\{ \frac{F \circ G}{F \in \operatorname{up} f, G \in \operatorname{up} g} \right\} \sqcup \bigsqcap^{\mathsf{RLD}} \left\{ \frac{F \circ H}{F \in \operatorname{up} f, H \in \operatorname{up} h} \right\} = \bigsqcap^{\mathsf{RLD}} \left\{ \frac{(F_1 \circ G) \sqcup^{\mathsf{RLD}} (F_2 \circ H)}{F_1, F_2 \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h} \right\} = \bigsqcap^{\mathsf{RLD}} \left\{ \frac{(F_1 \circ G) \sqcup (F_2 \circ H)}{F_1, F_2 \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h} \right\}.$$

Obviously

$$\bigsqcap^{\mathsf{RLD}} \left\{ \frac{(F_1 \circ G) \sqcup (F_2 \circ H)}{F_1, F_2 \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h} \right\} \sqsupseteq \bigsqcap^{\mathsf{RLD}} \left\{ \frac{(((F_1 \sqcap F_2) \circ G) \sqcup ((F_1 \sqcap F_2) \circ H))}{F_1, F_2 \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h} \right\} = \bigsqcap^{\mathsf{RLD}} \left\{ \frac{(F \circ G) \sqcup (F \circ H)}{F \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h} \right\} = \bigsqcap^{\mathsf{RLD}} \left\{ \frac{F \circ (G \sqcup H)}{F \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h} \right\}.$$

Because $G \in \operatorname{up} g \wedge H \in \operatorname{up} h \Rightarrow G \sqcup H \in \operatorname{up}(g \sqcup h)$ we have

$$\bigsqcap^{\mathsf{RLD}} \left\{ \frac{F \circ (G \sqcup H)}{F \in \operatorname{up} f, G \in \operatorname{up} g, H \in \operatorname{up} h} \right\} \sqsupseteq \bigsqcap^{\mathsf{RLD}} \left\{ \frac{F \circ K}{F \in \operatorname{up} f, K \in \operatorname{up}(g \sqcup h)} \right\} = f \circ (g \sqcup h).$$

Thus we have proved $f \circ g \sqcup f \circ h \sqsupseteq f \circ (g \sqcup h)$. But obviously $f \circ (g \sqcup h) \sqsupseteq f \circ g$ and $f \circ (g \sqcup h) \sqsupseteq f \circ h$ and so $f \circ (g \sqcup h) \sqsupseteq f \circ g \sqcup f \circ h$. Thus $f \circ (g \sqcup h) = f \circ g \sqcup f \circ h$. □

THEOREM 1006. Let A, B, C be sets, $f \in \mathsf{RLD}(A, B)$, $g \in \mathsf{RLD}(B, C)$, $h \in \mathsf{RLD}(A, C)$. Then

$$g \circ f \not\asymp h \Leftrightarrow g \not\asymp h \circ f^{-1}.$$