PROOF. Equivalently transform the defining formula for regular funcoids:
$\langle f\rangle\langle f^{-1}\rangle C \asymp \langle f\rangle@\{p\} \Leftarrow\uparrow^{\operatorname{Src} f} \{p\} \asymp \langle f^{-1}\rangle C$;
$\langle f\rangle\langle f^{-1}\rangle C \not\asymp \langle f\rangle@\{p\} \Rightarrow\uparrow^{\operatorname{Src} f} \{p\} \not\asymp \langle f^{-1}\rangle C$;
(by definition of funcoids)
$C \not\asymp \langle f\rangle\langle f^{-1}\rangle\langle f\rangle@\{p\} \Rightarrow C \not\asymp \langle f\rangle@\{p\}$;
$\langle f\rangle\langle f^{-1}\rangle\langle f\rangle@\{p\} \sqsubseteq \langle f\rangle@\{p\}$;
$\langle f \circ f^{-1} \circ f\rangle@\{p\} \sqsubseteq \langle f\rangle@\{p\}$;
$\operatorname{Compl}(f \circ f^{-1} \circ f) \sqsubseteq \operatorname{Compl} f$;
$\operatorname{Compl}(f \circ f^{-1} \circ f) \sqsubseteq f$. □

PROPOSITION 975. If f is complete, regularity of funcoid f is equivalent to $f \circ \operatorname{Compl}(f^{-1} \circ f) \sqsubseteq f$.

PROOF. By proposition 951. □

REMARK 976. After seeing how it collapses into algebraic formulas about funcoids, the definition for a funcoid being regular seems quite arbitrary and sucked out of the finger (not an example of algebraic elegance). So I present these formulas only because they coincide with the traditional definition of regular topological spaces. However this is only my personal opinion and it may be wrong.

DEFINITION 977. An endofuncoid is T_3- iff it is both T_2- and regular.

A topological space S is called T_4-separable when for any two disjoint closed sets $A, B \subseteq S$ there exist disjoint open sets U, V containing A and B respectively.

Let f be the complete funcoid corresponding to the topological space.

Since the closed sets are exactly sets of the form $\langle f^{-1}\rangle^* X$ and sets X and Y having non-intersecting open neighborhood is equivalent to $\langle f\rangle^* X \asymp \langle f\rangle^* Y$, the above is equivalent to:
$\langle f^{-1}\rangle^* A \asymp \langle f^{-1}\rangle^* B \Rightarrow \langle f\rangle^*\langle f^{-1}\rangle^* A \asymp \langle f\rangle^*\langle f^{-1}\rangle^* B$;
$\langle f\rangle^*\langle f^{-1}\rangle^* A \not\asymp \langle f\rangle^*\langle f^{-1}\rangle^* B \Rightarrow \langle f^{-1}\rangle^* A \not\asymp \langle f^{-1}\rangle^* B$;
$\langle f\rangle^*\langle f^{-1}\rangle^*\langle f\rangle^*\langle f^{-1}\rangle^* A \not\asymp B \Rightarrow \langle f\rangle^*\langle f^{-1}\rangle^* A \not\asymp B$;
$\langle f\rangle^*\langle f^{-1}\rangle^*\langle f\rangle^*\langle f^{-1}\rangle^* A \sqsubseteq \langle f\rangle^*\langle f^{-1}\rangle^* A$;
$f \circ f^{-1} \circ f \circ f^{-1} \sqsubseteq f \circ f^{-1}$.
Take the last formula as the definition of T_4-funcoid f.

7.18. Filters closed regarding a funcoid

DEFINITION 978. Let's call *closed* regarding a funcoid $f \in \mathsf{FCD}(A, A)$ such filter $\mathcal{A} \in \mathscr{F}(\operatorname{Src} f)$ that $\langle f\rangle\mathcal{A} \sqsubseteq \mathcal{A}$.

This is a generalization of closedness of a set regarding an unary operation.

PROPOSITION 979. If I and J are closed (regarding some funcoid f), S is a set of closed filters on $\operatorname{Src} f$, then

1°. $\mathcal{I} \sqcup \mathcal{J}$ is a closed filter;
2°. $\bigsqcap S$ is a closed filter.

PROOF. Let denote the given funcoid as f. $\langle f\rangle(\mathcal{I} \sqcup \mathcal{J}) = \langle f\rangle\mathcal{I} \sqcup \langle f\rangle\mathcal{J} \sqsubseteq \mathcal{I} \sqcup \mathcal{J}$, $\langle f\rangle \bigsqcap S \sqsubseteq \bigsqcap\langle\langle f\rangle\rangle^* S \sqsubseteq \bigsqcap S$. Consequently the filters $\mathcal{I} \sqcup \mathcal{J}$ and $\bigsqcap S$ are closed. □

PROPOSITION 980. If S is a set of filters closed regarding a complete funcoid, then the filter $\bigsqcup S$ is also closed regarding our funcoid.

PROOF. $\langle f\rangle \bigsqcup S = \bigsqcup\langle\langle f\rangle\rangle^* S \sqsubseteq \bigsqcup S$ where f is the given funcoid. □