PROOF.

$$\begin{aligned}&\langle g\circ f\rangle\langle\mu\rangle^*@\{x\} =\\ &\langle g\rangle\langle f\rangle\langle\mu\rangle^*@\{x\} \sqsupseteq\\ &\langle g\rangle\langle\nu\rangle\langle f\rangle^*@\{x\} \sqsupseteq \quad \text{(using that } f \text{ is monovalued and principal)}\\ &\langle\pi\rangle\langle g\rangle\langle f\rangle^*@\{x\} =\\ &\langle\pi\rangle\langle g\circ f\rangle@\{x\}.\end{aligned}$$

□

PROBLEM 966. Devise a pointfree (not using a particular point x) proof of the above theorem. It should refer to a lemma which may use a particular point, but the proof of the theorem itself should be without a particular point.

7.17. T_0-, T_1-, T_2-, T_3-, and T_4-separable funcoids

For funcoids it can be generalized T_0-, T_1-, T_2-, and T_3- separability. Worthwhile note that T_0 and T_2 separability is defined through T_1 separability.

DEFINITION 967. Let call T_1*-separable* such endofuncoid f that for every $\alpha,\beta\in\operatorname{Ob} f$ is true

$$\alpha\neq\beta\Rightarrow\neg(@\{\alpha\}\ [f]^*\ @\{\beta\}).$$

PROPOSITION 968. An endofuncoid f is T_1-separable iff $\operatorname{Cor} f\sqsubseteq 1^{\mathsf{FCD}}_{\operatorname{Ob} f}$.

PROOF.

$$\forall x,y\in\operatorname{Ob} f:(@\{x\}\ [f]^*\ @\{y\}\Rightarrow x=y)\Leftrightarrow\\ \forall x,y\in\operatorname{Ob} f:(@\{x\}\ [\operatorname{Cor} f]^*\ @\{y\}\Rightarrow x=y)\Leftrightarrow\operatorname{Cor} f\sqsubseteq 1^{\mathsf{FCD}}_{\operatorname{Ob} f}.$$

□

PROPOSITION 969. An endofuncoid f is T_1-separable iff $\operatorname{Cor}\langle f\rangle^*\{x\}\sqsubseteq\{x\}$ for every $x\in\operatorname{Ob} f$.

PROOF. $\operatorname{Cor}\langle f\rangle^*\{x\}\sqsubseteq\{x\}\Leftrightarrow\langle\operatorname{CoCompl} f\rangle^*\{x\}\sqsubseteq\{x\}\Leftrightarrow\operatorname{Compl}\operatorname{CoCompl} f\sqsubseteq 1^{\mathsf{FCD}}_{\operatorname{Ob} f}\Leftrightarrow\operatorname{Cor} f\sqsubseteq 1^{\mathsf{FCD}}_{\operatorname{Ob} f}$. □

DEFINITION 970. Let call T_0*-separable* such funcoid $f\in\mathsf{FCD}(A,A)$ that $f\sqcap f^{-1}$ is T_1-separable.

DEFINITION 971. Let call T_2*-separable* such funcoid f that $f^{-1}\circ f$ is T_1-separable.

For symmetric transitive funcoids T_0-, T_1- and T_2-separability are the same (see theorem 252).

OBVIOUS 972. A funcoid f is T_2-separable iff $\alpha\neq\beta\Rightarrow\langle f\rangle^*@\{\alpha\}\not\asymp\langle f\rangle^*@\{\beta\}$ for every $\alpha,\beta\in\operatorname{Src} f$.

DEFINITION 973. Funcoid f is *regular* iff for every $C\in\mathscr{T}\operatorname{Dst} f$ and $p\in\operatorname{Src} f$

$$\langle f\rangle\langle f^{-1}\rangle C\asymp\langle f\rangle@\{p\}\Leftarrow\uparrow^{\operatorname{Src} f}\{p\}\asymp\langle f^{-1}\rangle C.$$

PROPOSITION 974. The following are pairwise equivalent:

1°. A funcoid f is regular.
2°. $\operatorname{Compl}(f\circ f^{-1}\circ f)\sqsubseteq\operatorname{Compl} f$.
3°. $\operatorname{Compl}(f\circ f^{-1}\circ f)\sqsubseteq f$.