$\langle f\circ f^{-1}\rangle p = \langle f\rangle\langle f^{-1}\rangle p \sqsupseteq \langle f\rangle a \sqsupseteq q$; $\langle f\circ f^{-1}\rangle p \not\sqsubseteq p$ and $\langle f\circ f^{-1}\rangle p \neq \bot^{\mathscr{F}(\operatorname{Dst} f)}$. So it cannot be $f\circ f^{-1}\sqsubseteq 1^{\mathsf{FCD}}_{\operatorname{Dst} f}$.

2°⇒3°. Obvious.

1°⇒2°.

$$\left\langle\left(\bigsqcap G\right)\circ f\right\rangle x = \left\langle\bigsqcap G\right\rangle\langle f\rangle x = \bigsqcap_{g\in G}\langle g\rangle\langle f\rangle x = \bigsqcap_{g\in G}\langle g\circ f\rangle x = \left\langle\bigsqcap_{g\in G}(g\circ f)\right\rangle x$$

for every atomic filter object $x\in\operatorname{atoms}^{\mathscr{F}(\operatorname{Src} f)}$. Thus $(\bigsqcap G)\circ f = \bigsqcap_{g\in G}(g\circ f)$.

3°⇒1°. Take $g = a\times^{\mathsf{FCD}} y$ and $h = b\times^{\mathsf{FCD}} y$ for arbitrary atomic filter objects $a\neq b$ and y. We have $g\sqcap h=\bot$; thus $(g\circ f)\sqcap(h\circ f) = (g\sqcap h)\circ f = \bot$ and thus impossible $x\,[f]\,a\wedge x\,[f]\,b$ as otherwise $x\,[g\circ f]\,y$ and $x\,[h\circ f]\,y$ so $x\,[(g\circ f)\sqcap(h\circ f)]\,y$. Thus f is monovalued.

□

Corollary 959. A binary relation corresponds to a monovalued funcoid iff it is a function.

Proof. Because $\forall I,J\in\mathscr{P}(\operatorname{im} f):\left\langle f^{-1}\right\rangle^*(I\sqcap J) = \left\langle f^{-1}\right\rangle^* I\sqcap\left\langle f^{-1}\right\rangle^* J$ is true for a funcoid f corresponding to a binary relation if and only if it is a function (see proposition 385). □

Remark 960. This corollary can be reformulated as follows: For binary relations (principal funcoids) the classic concept of monovaluedness and monovaluedness in the above defined sense of monovaluedness of a funcoid are the same.

Theorem 961. If f, g are funcoids, $f\sqsubseteq g$ and g is monovalued then $g|_{\operatorname{dom} f} = f$.

Proof. Obviously $g|_{\operatorname{dom} f}\sqsupseteq f$. Suppose for contrary that $g|_{\operatorname{dom} f}\sqsubset f$. Then there exists an atom $a\in\operatorname{atoms}\operatorname{dom} f$ such that $\langle g|_{\operatorname{dom} f}\rangle a\neq\langle f\rangle a$ that is $\langle g\rangle a\sqsubset\langle f\rangle a$ what is impossible. □

7.16. Open maps

Definition 962. An *open map* from a topological space to a topological space is a function which maps open sets into open sets.

An obvious generalization of this is *open map* f from an endofuncoid μ to an endofuncoid ν, which is by definition a function (or rather a principal, entirely defined, monovalued funcoid) from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\nu$ such that

$$\forall x\in\operatorname{Ob}\mu, V\in\langle\mu\rangle^*\{x\}:\langle f\rangle^* V\sqsupseteq\langle\nu\rangle\langle f\rangle^*@\{x\}.$$

This formula is equivalent (exercise!) to

$$\forall x\in\operatorname{Ob}\mu:\langle f\rangle\langle\mu\rangle^*@\{x\}\sqsupseteq\langle\nu\rangle\langle f\rangle^*@\{x\}.$$

It can be abstracted/simplified further (now for an *arbitrary* funcoid f from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\nu$):

$$\operatorname{Compl}(f\circ\mu)\sqsupseteq\operatorname{Compl}(\nu\circ f).$$

Definition 963. An *open funcoid* from an endofuncoid μ to an endofuncoid ν is a funcoid f from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\nu$ such that $\operatorname{Compl}(f\circ\mu)\sqsupseteq\operatorname{Compl}(\nu\circ f)$.

Obvious 964. A funcoid f is open iff $f\circ\mu\sqsupseteq\operatorname{Compl}(\nu\circ f)$.

Theorem 965. Let μ, ν, π be endofuncoids. Let f be an principal monovalued open funcoid from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\nu$ and g is a open funcoid from $\operatorname{Ob}\nu$ to $\operatorname{Ob}\pi$. Then $g\circ f$ is an open funcoid from $\operatorname{Ob}\mu$ to $\operatorname{Ob}\pi$.