In other words, a funcoid is monovalued (injective) when it is a monovalued (injective) morphism of the category of funcoids. Monovaluedness is dual of injectivity.

OBVIOUS 957.

1°. A morphism $(\mathcal{A}, \mathcal{B}, f)$ of the category of funcoid triples is monovalued iff the funcoid f is monovalued.
2°. A morphism $(\mathcal{A}, \mathcal{B}, f)$ of the category of funcoid triples is injective iff the funcoid f is injective.

THEOREM 958. The following statements are equivalent for a funcoid f:

1°. f is monovalued.
2°. It is metamonovalued.
3°. It is weakly metamonovalued.
4°. $\forall a \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Src} f)} : \langle f\rangle a \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Dst} f)} \cup \{\bot^{\mathscr{F}(\operatorname{Dst} f)}\}$.
5°. $\forall \mathcal{I}, \mathcal{J} \in \mathscr{F}(\operatorname{Dst} f) : \langle f^{-1}\rangle(\mathcal{I} \sqcap \mathcal{J}) = \langle f^{-1}\rangle\mathcal{I} \sqcap \langle f^{-1}\rangle\mathcal{J}$.
6°. $\forall I, J \in \mathscr{T}(\operatorname{Dst} f) : \langle f^{-1}\rangle^*(I \sqcap J) = \langle f^{-1}\rangle^* I \sqcap \langle f^{-1}\rangle^* J$.

PROOF.

4°⇒5°. Let $a \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Src} f)}$, $\langle f\rangle a = b$. Then because $b \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Dst} f)} \cup \{\bot^{\mathscr{F}(\operatorname{Dst} f)}\}$

$$(\mathcal{I} \sqcap \mathcal{J}) \sqcap b \neq \bot \Leftrightarrow \mathcal{I} \sqcap b \neq \bot \wedge \mathcal{J} \sqcap b \neq \bot;$$
$$a\ [f]\ \mathcal{I} \sqcap \mathcal{J} \Leftrightarrow a\ [f]\ \mathcal{I} \wedge a\ [f]\ \mathcal{J};$$
$$\mathcal{I} \sqcap \mathcal{J}\ \left[f^{-1}\right]\ a \Leftrightarrow \mathcal{I}\ \left[f^{-1}\right]\ a \wedge \mathcal{J}\ \left[f^{-1}\right]\ a;$$
$$a \sqcap \langle f^{-1}\rangle(\mathcal{I} \sqcap \mathcal{J}) \neq \bot \Leftrightarrow a \sqcap \langle f^{-1}\rangle\mathcal{I} \neq \bot \wedge a \sqcap \langle f^{-1}\rangle\mathcal{J} \neq \bot;$$
$$\langle f^{-1}\rangle(\mathcal{I} \sqcap \mathcal{J}) = \langle f^{-1}\rangle\mathcal{I} \sqcap \langle f^{-1}\rangle\mathcal{J}.$$

5°⇒1°. $\langle f^{-1}\rangle a \sqcap \langle f^{-1}\rangle b = \langle f^{-1}\rangle(a \sqcap b) = \langle f^{-1}\rangle\bot = \bot$ for every two distinct atomic filter objects a and b on $\operatorname{Dst} f$. This is equivalent to $\neg(\langle f^{-1}\rangle a\ [f]\ b)$; $b \asymp \langle f\rangle\langle f^{-1}\rangle a$; $b \asymp \langle f \circ f^{-1}\rangle a$; $\neg(a\ \left[f \circ f^{-1}\right]\ b)$. So $a\ \left[f \circ f^{-1}\right]\ b \Rightarrow a = b$ for every ultrafilters a and b. This is possible only when $f \circ f^{-1} \sqsubseteq 1^{\mathsf{FCD}}_{\operatorname{Dst} f}$.

6°⇒5°.

$$\begin{aligned}
\langle f^{-1}\rangle(\mathcal{I} \sqcap \mathcal{J}) &= \\
\bigsqcap \langle \langle f\rangle^* \rangle^* \operatorname{up}(\mathcal{I} \sqcap \mathcal{J}) &= \\
\bigsqcap \langle \langle f\rangle^* \rangle^* \left\{ \frac{I \sqcap J}{I \in \operatorname{up}\mathcal{I}, J \in \operatorname{up}\mathcal{J}} \right\} &= \\
\bigsqcap \left\{ \frac{\langle f\rangle^*(I \sqcap J)}{I \in \operatorname{up}\mathcal{I}, J \in \operatorname{up}\mathcal{J}} \right\} &= \\
\bigsqcap \left\{ \frac{\langle f\rangle^* I \sqcap \langle f\rangle^* J}{I \in \operatorname{up}\mathcal{I}, J \in \operatorname{up}\mathcal{J}} \right\} &= \\
\bigsqcap \left\{ \frac{\langle f\rangle^* I}{I \in \operatorname{up}\mathcal{I}} \right\} \sqcap \bigsqcap \left\{ \frac{\langle f\rangle^* J}{J \in \operatorname{up}\mathcal{J}} \right\} &= \\
\langle f^{-1}\rangle\mathcal{I} \sqcap \langle f^{-1}\rangle\mathcal{J}.
\end{aligned}$$

5°⇒6°. Obvious.

¬4°⇒¬1°. Suppose $\langle f\rangle a \notin \operatorname{atoms}^{\mathscr{F}(\operatorname{Dst} f)} \cup \{\bot^{\mathscr{F}(\operatorname{Dst} f)}\}$ for some $a \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Src} f)}$. Then there exist two atomic filters p and q on $\operatorname{Dst} f$ such that $p \neq q$ and $\langle f\rangle a \sqsupseteq p \wedge \langle f\rangle a \sqsupseteq q$. Consequently $p \not\asymp \langle f\rangle a$; $a \not\asymp \langle f^{-1}\rangle p$; $a \sqsubseteq \langle f^{-1}\rangle p$;