

2°. $(\text{CoCompl}(g \circ f))^{-1} = f^{-1} \circ (\text{CoCompl } g)^{-1}$; $\text{Compl}(g \circ f)^{-1} = f^{-1} \circ \text{Compl } g^{-1}$; $\text{Compl}(f^{-1} \circ g^{-1}) = f^{-1} \circ \text{Compl } g^{-1}$. After variable replacement we get $\text{Compl}(f \circ g) = f \circ \text{Compl } g$ (after the replacement f is a complete functor). \square

COROLLARY 952. For every composable functors f and g

- 1°. $\text{Compl } f \circ \text{Compl } g = \text{Compl}(\text{Compl } f \circ g)$.
- 2°. $\text{CoCompl } g \circ \text{CoCompl } f = \text{CoCompl}(g \circ \text{CoCompl } f)$.

PROPOSITION 953. For every composable functors f and g

- 1°. $\text{Compl}(g \circ f) = \text{Compl}(g \circ (\text{Compl } f))$;
- 2°. $\text{CoCompl}(g \circ f) = \text{CoCompl}((\text{CoCompl } g) \circ f)$.

PROOF.

1°.

$$\begin{aligned} \langle g \circ (\text{Compl } f) \rangle^* @ \{x\} &= \langle g \rangle \langle \text{Compl } f \rangle^* @ \{x\} = \\ &= \langle g \rangle \langle f \rangle^* @ \{x\} = \langle g \circ f \rangle^* @ \{x\}. \end{aligned}$$

Thus $\text{Compl}(g \circ (\text{Compl } f)) = \text{Compl}(g \circ f)$.

2°. $(\text{Compl}(g \circ (\text{Compl } f)))^{-1} = (\text{Compl}(g \circ f))^{-1}$; $\text{CoCompl}(g \circ (\text{Compl } f))^{-1} = \text{CoCompl}(g \circ f)^{-1}$; $\text{CoCompl}((\text{Compl } f)^{-1} \circ g^{-1}) = \text{CoCompl}(f^{-1} \circ g^{-1})$; $\text{CoCompl}((\text{CoCompl } f^{-1}) \circ g^{-1}) = \text{CoCompl}(f^{-1} \circ g^{-1})$. After variable replacement $\text{CoCompl}((\text{CoCompl } g) \circ f) = \text{CoCompl}(g \circ f)$. \square

THEOREM 954. The filtrator of functors (from a given set A to a given set B) is with co-separable core.

PROOF. Let $f, g \in \text{FCD}(A, B)$ and $f \sqcup g = \top$. Then for every $X \in \mathcal{T}A$ we have

$$\begin{aligned} \langle f \rangle^* X \sqcup \langle g \rangle^* X = \top &\Leftrightarrow \text{Cor} \langle f \rangle^* X \sqcup \text{Cor} \langle g \rangle^* X = \top \Leftrightarrow \\ &\langle \text{CoCompl } f \rangle^* X \sqcup \langle \text{CoCompl } g \rangle^* X = \top. \end{aligned}$$

Thus $\langle \text{CoCompl } f \sqcup \text{CoCompl } g \rangle^* X = \top$;

$$f \sqcup g = \top \Rightarrow \text{CoCompl } f \sqcup \text{CoCompl } g = \top. \quad (14)$$

Applying the dual of the formulas (14) to the formula (14) we get:

$$f \sqcup g = \top \Rightarrow \text{Compl } \text{CoCompl } f \sqcup \text{Compl } \text{CoCompl } g = \top$$

that is $f \sqcup g = \top \Rightarrow \text{Cor } f \sqcup \text{Cor } g = \top$. So $\text{FCD}(A, B)$ is with co-separable core. \square

COROLLARY 955. The filtrator of complete functors is also with co-separable core.

7.15. Monovalued and injective functors

Following the idea of definition of monovalued morphism let's call *monovalued* such a functor f that $f \circ f^{-1} \sqsubseteq \text{id}_{\text{im } f}^{\text{FCD}}$.

Similarly, I will call a functor injective when $f^{-1} \circ f \sqsubseteq \text{id}_{\text{dom } f}^{\text{FCD}}$.

OBVIOUS 956. A functor f is:

- 1°. monovalued iff $f \circ f^{-1} \sqsubseteq 1_{\text{Dst } f}^{\text{FCD}}$;
- 2°. injective iff $f^{-1} \circ f \sqsubseteq 1_{\text{Src } f}^{\text{FCD}}$.