PROOF. Let denote $R$ the right part of the equality to prove.

$\langle R\rangle^* @\{\beta\} = \bigsqcup_{\alpha\in\operatorname{Src} f} \langle f|_{\uparrow\{\alpha\}}\rangle^* @\{\beta\} = \langle f\rangle^* @\{\beta\}$ for every $\beta \in \operatorname{Src} f$ and $R$ is complete as a join of complete funcoids.

Thus $R$ is the completion of $f$. □

CONJECTURE 947. $\operatorname{Compl} f = f \setminus^* (\Omega \times^{\mathsf{FCD}} \mho)$.

This conjecture may be proved by considerations similar to these in the section "Fréchet filter".

LEMMA 948. Co-completion of a complete funcoid is complete.

PROOF. Let $f$ be a complete funcoid.

$$\langle \operatorname{CoCompl} f\rangle^* X = \operatorname{Cor}\langle f\rangle^* X = \operatorname{Cor} \bigsqcup_{x\in\operatorname{atoms} X} \langle f\rangle^* x = \bigsqcup_{x\in\operatorname{atoms} X} \operatorname{Cor}\langle f\rangle^* x = \bigsqcup_{x\in\operatorname{atoms} X} \langle \operatorname{CoCompl} f\rangle^* x$$

for every set typed $X \in \mathscr{T}(\operatorname{Src} f)$. Thus $\operatorname{CoCompl} f$ is complete. □

THEOREM 949. $\operatorname{Compl}\operatorname{CoCompl} f = \operatorname{CoCompl}\operatorname{Compl} f = \operatorname{Cor} f$ for every funcoid $f$.

PROOF. $\operatorname{Compl}\operatorname{CoCompl} f$ is co-complete since (used the lemma) $\operatorname{CoCompl} f$ is co-complete. Thus $\operatorname{Compl}\operatorname{CoCompl} f$ is a principal funcoid. $\operatorname{CoCompl} f$ is the greatest co-complete funcoid under $f$ and $\operatorname{Compl}\operatorname{CoCompl} f$ is the greatest complete funcoid under $\operatorname{CoCompl} f$. So $\operatorname{Compl}\operatorname{CoCompl} f$ is greater than any principal funcoid under $\operatorname{CoCompl} f$ which is greater than any principal funcoid under $f$. Thus $\operatorname{Compl}\operatorname{CoCompl} f$ is the greatest principal funcoid under $f$. Thus $\operatorname{Compl}\operatorname{CoCompl} f = \operatorname{Cor} f$. Similarly $\operatorname{CoCompl}\operatorname{Compl} f = \operatorname{Cor} f$. □

### 7.14.1. More on completion of funcoids.

PROPOSITION 950. For every composable funcoids $f$ and $g$

1°. $\operatorname{Compl}(g\circ f) \sqsupseteq \operatorname{Compl} g \circ \operatorname{Compl} f$;
2°. $\operatorname{CoCompl}(g\circ f) \sqsupseteq \operatorname{CoCompl} g \circ \operatorname{CoCompl} f$.

PROOF.

1°. $\operatorname{Compl} g \circ \operatorname{Compl} f = \operatorname{Compl}(\operatorname{Compl} g \circ \operatorname{Compl} f) \sqsubseteq \operatorname{Compl}(g\circ f)$.
2°. $\operatorname{CoCompl} g \circ \operatorname{CoCompl} f = \operatorname{CoCompl}(\operatorname{CoCompl} g \circ \operatorname{CoCompl} f) \sqsubseteq \operatorname{CoCompl}(g\circ f)$.

□

PROPOSITION 951. For every composable funcoids $f$ and $g$

1°. $\operatorname{CoCompl}(g\circ f) = (\operatorname{CoCompl} g)\circ f$ if $f$ is a co-complete funcoid.
2°. $\operatorname{Compl}(f\circ g) = f\circ \operatorname{Compl} g$ if $f$ is a complete funcoid.

PROOF.

1°. For every $X \in \mathscr{T}(\operatorname{Src} f)$

$$\begin{aligned}\langle \operatorname{CoCompl}(g\circ f)\rangle^* X &= \\ \operatorname{Cor}\langle g\circ f\rangle^* X &= \\ \operatorname{Cor}\langle g\rangle\langle f\rangle^* X &= \\ \langle \operatorname{CoCompl} g\rangle\langle f\rangle^* X &= \\ \langle(\operatorname{CoCompl} g)\circ f\rangle^* X&.\end{aligned}$$