□

THEOREM 939. $\langle \operatorname{CoCompl} f\rangle^* X = \operatorname{Cor}\langle f\rangle^* X$ for every funcoid f and typed set $X \in \mathscr{T}(\operatorname{Src} f)$.

PROOF. $\operatorname{CoCompl} f \sqsubseteq f$ thus $\langle \operatorname{CoCompl} f\rangle^* X \sqsubseteq \langle f\rangle^* X$ but $\langle \operatorname{CoCompl} f\rangle^* X$ is a principal filter thus $\langle \operatorname{CoCompl} f\rangle^* X \sqsubseteq \operatorname{Cor}\langle f\rangle^* X$.

Let $\alpha X = \operatorname{Cor}\langle f\rangle^* X$. Then $\alpha \bot^{\mathscr{T}(\operatorname{Src} f)} = \bot^{\mathscr{F}(\operatorname{Dst} f)}$ and

$$\alpha(X \sqcup Y) = \operatorname{Cor}\langle f\rangle^*(X \sqcup Y) = \operatorname{Cor}(\langle f\rangle^* X \sqcup \langle f\rangle^* Y) = \operatorname{Cor}\langle f\rangle^* X \sqcup \operatorname{Cor}\langle f\rangle^* Y = \alpha X \sqcup \alpha Y$$

(used theorem 600). Thus α can be continued till $\langle g\rangle$ for some funcoid g. This funcoid is co-complete.

Evidently g is the greatest co-complete element of $\mathsf{FCD}(\operatorname{Src} f, \operatorname{Dst} f)$ which is lower than f.

Thus $g = \operatorname{CoCompl} f$ and $\operatorname{Cor}\langle f\rangle^* X = \alpha X = \langle g\rangle^* X = \langle \operatorname{CoCompl} f\rangle^* X$. □

THEOREM 940. $\mathsf{ComplFCD}(A, B)$ is an atomistic lattice.

PROOF. Let $f \in \mathsf{ComplFCD}(A, B)$, $X \in \mathscr{T}(\operatorname{Src} f)$.

$$\langle f\rangle^* X = \bigsqcup_{x \in \operatorname{atoms} X} \langle f\rangle^* x = \bigsqcup_{x \in \operatorname{atoms} X} \langle f|_x\rangle^* x = \bigsqcup_{x \in \operatorname{atoms} X} \langle f|_x\rangle^* X,$$

thus $f = \bigsqcup_{x \in \operatorname{atoms} X}(f|_x)$. It is trivial that every $f|_x$ is a join of atoms of $\mathsf{ComplFCD}(A, B)$. □

THEOREM 941. A funcoid is complete iff it is a join (on the lattice $\mathsf{FCD}(A, B)$) of atomic complete funcoids.

PROOF. It follows from the theorem 919 and the previous theorem. □

COROLLARY 942. $\mathsf{ComplFCD}(A, B)$ is join-closed.

THEOREM 943. $\operatorname{Compl} \bigsqcup R = \bigsqcup \langle \operatorname{Compl}\rangle^* R$ for every $R \in \mathscr{P}\mathsf{FCD}(A, B)$ (for every sets A, B).

PROOF. For every typed set X

$$\begin{aligned}
&\left\langle \operatorname{Compl} \bigsqcup R\right\rangle^* X = \\
&\bigsqcup_{x \in \operatorname{atoms} X} \left\langle \bigsqcup R\right\rangle^* x = \\
&\bigsqcup_{x \in \operatorname{atoms} X} \bigsqcup_{f \in R} \langle f\rangle^* x = \\
&\bigsqcup_{f \in R} \bigsqcup_{x \in \operatorname{atoms} X} \langle f\rangle^* x = \\
&\bigsqcup_{f \in R} \langle \operatorname{Compl} f\rangle^* X = \\
&\left\langle \bigsqcup \langle \operatorname{Compl}\rangle^* R\right\rangle^* X.
\end{aligned}$$

□

COROLLARY 944. Compl is a lower adjoint.

CONJECTURE 945. Compl is not an upper adjoint (in general).

PROPOSITION 946. $\operatorname{Compl} f = \bigsqcup_{\alpha \in \operatorname{Src} f}(f|_{\uparrow\{\alpha\}})$ for every funcoid f.