1°. $\alpha\bot = \bot$;
2°. $\forall I, J \in \mathscr{T} A : \alpha(I \sqcup J) = \alpha I \sqcup \alpha J$.

OBVIOUS 913. A funcoid f is co-complete iff $\langle f\rangle^* = \uparrow \circ \alpha$ for a generalized closure α.

REMARK 914. Thus funcoids can be considered as a generalization of generalized closures. A topological space in Kuratowski sense is the same as reflexive and transitive generalized closure. So topological spaces can be considered as a special case of funcoids.

DEFINITION 915. I will call a *complete funcoid* a funcoid whose reverse is co-complete.

THEOREM 916. The following conditions are equivalent for every funcoid f:
1°. funcoid f is complete;
2°. $\forall S \in \mathscr{P}\mathscr{F}(\operatorname{Src} f), J \in \mathscr{T}(\operatorname{Dst} f) : (\bigsqcup S\ [f]\ J \Leftrightarrow \exists \mathcal{I} \in S : \mathcal{I}\ [f]\ J)$;
3°. $\forall S \in \mathscr{P}\mathscr{T}(\operatorname{Src} f), J \in \mathscr{T}(\operatorname{Dst} f) : \left(\bigsqcup S\ [f]^*\ J \Leftrightarrow \exists I \in S : I\ [f]^*\ J\right)$;
4°. $\forall S \in \mathscr{P}\mathscr{F}(\operatorname{Src} f) : \langle f\rangle \bigsqcup S = \bigsqcup \langle\langle f\rangle\rangle^* S$;
5°. $\forall S \in \mathscr{P}\mathscr{T}(\operatorname{Src} f) : \langle f\rangle^* \bigsqcup S = \bigsqcup \langle \langle f\rangle^*\rangle^* S$;
6°. $\forall A \in \mathscr{T}(\operatorname{Src} f) : \langle f\rangle^* A = \bigsqcup_{a \in \operatorname{atoms} A} \langle f\rangle^* a$.

PROOF.

3°⇒1°. For every $S \in \mathscr{P}\mathscr{T}(\operatorname{Src} f)$, $J \in \mathscr{T}(\operatorname{Dst} f)$

$$\bigsqcup S \sqcap \left\langle f^{-1}\right\rangle^* J \neq \bot \Leftrightarrow \exists I \in S : I \sqcap \left\langle f^{-1}\right\rangle^* J \neq \bot,$$

consequently by theorem 580 we have that $\left\langle f^{-1}\right\rangle^* J$ is a principal filter.

1°⇒2°. For every $S \in \mathscr{P}\mathscr{F}(\operatorname{Src} f)$, $J \in \mathscr{T}(\operatorname{Dst} f)$ we have that $\left\langle f^{-1}\right\rangle^* J$ is a principal filter, consequently

$$\bigsqcup S \sqcap \left\langle f^{-1}\right\rangle^* J \neq \bot \Leftrightarrow \exists \mathcal{I} \in S : \mathcal{I} \sqcap \left\langle f^{-1}\right\rangle^* J \neq \bot.$$

From this follows 2°.

6°⇒5°.

$$\begin{gathered}\langle f\rangle^* \bigsqcup S = \\ \bigsqcup_{a \in \operatorname{atoms} \bigsqcup S} \langle f\rangle^* a = \\ \bigsqcup \bigcup_{A \in S} \left\{ \frac{\langle f\rangle^* a}{a \in \operatorname{atoms} A} \right\} = \\ \bigsqcup_{A \in S} \bigsqcup_{a \in \operatorname{atoms} A} \langle f\rangle^* a = \\ \bigsqcup_{A \in S} \langle f\rangle^* A = \\ \bigsqcup \langle \langle f\rangle^*\rangle^* S.\end{gathered}$$

2°⇒4°. Using theorem 580,

$$\begin{gathered}J \not\asymp \langle f\rangle \bigsqcup S \Leftrightarrow \\ \bigsqcup S\ [f]\ J \Leftrightarrow \\ \exists \mathcal{I} \in S : \mathcal{I}\ [f]\ J \Leftrightarrow \\ \exists \mathcal{I} \in S : J \not\asymp \langle f\rangle \mathcal{I} \Leftrightarrow \\ J \not\asymp \bigsqcup \langle\langle f\rangle\rangle^* S.\end{gathered}$$