PROOF. $a \times^{\mathsf{FCD}} b \not\asymp f \sqcup g \Leftrightarrow a \; [f \sqcup g] \; b \Leftrightarrow a \; [f] \; b \vee a \; [g] \; b \Leftrightarrow a \times^{\mathsf{FCD}} b \not\asymp f \vee a \times^{\mathsf{FCD}} b \not\asymp g$ for every atomic filters a and b. □

THEOREM 902. The set of funcoids between sets A and B is a co-frame.

PROOF. Theorems 828 and 530. □

REMARK 903. The above proof does not use axiom of choice (unlike the below proof).

See also an older proof of the set of funcoids being co-brouwerian:

THEOREM 904. For every $f, g, h \in \mathsf{FCD}(A, B)$, $R \in \mathscr{P}\mathsf{FCD}(A, B)$ (for every sets A and B)

1°. $f \sqcap (g \sqcup h) = (f \sqcap g) \sqcup (f \sqcap h)$;
2°. $f \sqcup \bigsqcap R = \bigsqcap \langle f \sqcup \rangle^* R$.

PROOF. We will take into account that the lattice of funcoids is an atomistic lattice.

1°.

$$\begin{aligned}&\operatorname{atoms}(f \sqcap (g \sqcup h)) =\\ &\operatorname{atoms} f \cap \operatorname{atoms}(g \sqcup h) =\\ &\operatorname{atoms} f \cap (\operatorname{atoms} g \cup \operatorname{atoms} h) =\\ &(\operatorname{atoms} f \cap \operatorname{atoms} g) \cup (\operatorname{atoms} f \cap \operatorname{atoms} h) =\\ &\operatorname{atoms}(f \sqcap g) \cup \operatorname{atoms}(f \sqcap h) =\\ &\operatorname{atoms}((f \sqcap g) \sqcup (f \sqcap h)).\end{aligned}$$

2°.

$$\begin{aligned}\operatorname{atoms}\left(f \sqcup \bigsqcap R\right) &=\\ \operatorname{atoms} f \cup \operatorname{atoms} \bigsqcap R &=\\ \operatorname{atoms} f \cup \bigcap \langle \operatorname{atoms} \rangle^* R &=\\ \bigcap \langle (\operatorname{atoms} f) \cup \rangle^* \langle \operatorname{atoms} \rangle^* R &= \text{(use the following equality)}\\ \bigcap \langle \operatorname{atoms} \rangle^* \langle f \sqcup \rangle^* R &=\\ \operatorname{atoms} \bigsqcap \langle f \sqcup \rangle^* R. &\end{aligned}$$

$$\begin{aligned}\langle (\operatorname{atoms} f) \cup \rangle^* \langle \operatorname{atoms} \rangle^* R &=\\ \left\{ \frac{(\operatorname{atoms} f) \cup A}{A \in \langle \operatorname{atoms} \rangle^* R} \right\} &=\\ \left\{ \frac{(\operatorname{atoms} f) \cup A}{\exists C \in R : A = \operatorname{atoms} C} \right\} &=\\ \left\{ \frac{(\operatorname{atoms} f) \cup (\operatorname{atoms} C)}{C \in R} \right\} &=\\ \left\{ \frac{\operatorname{atoms}(f \sqcup C)}{C \in R} \right\} &=\\ \left\{ \frac{\operatorname{atoms} B}{\exists C \in R : B = f \sqcup C} \right\} &=\\ \left\{ \frac{\operatorname{atoms} B}{B \in \langle f \sqcup \rangle^* C} \right\} &=\\ \langle \operatorname{atoms} \rangle^* \langle f \sqcup \rangle^* R. &\end{aligned}$$