COROLLARY 885. $f|_{\mathcal{A}} = f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \top^{\mathscr{F}(\operatorname{Dst} f)})$ for every funcoid f and $\mathcal{A} \in \mathscr{F}(\operatorname{Src} f)$.

PROOF. $f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \top^{\mathscr{F}(\operatorname{Dst} f)}) = \operatorname{id}^{\mathsf{FCD}}_{\top^{\mathscr{F}(\operatorname{Dst} f)}} \circ f \circ \operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}} = f \circ \operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}} = f|_{\mathcal{A}}$. □

COROLLARY 886. $f \not\asymp \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \Leftrightarrow \mathcal{A}\ [f]\ \mathcal{B}$ for every funcoid f and $\mathcal{A} \in \mathscr{F}(\operatorname{Src} f)$, $\mathcal{B} \in \mathscr{F}(\operatorname{Dst} f)$.

PROOF.

$$\begin{aligned}
&f \not\asymp \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \Leftrightarrow\\
&\langle f \sqcap (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B})\rangle^* \top \neq \bot \Leftrightarrow\\
&\left\langle \operatorname{id}^{\mathsf{FCD}}_{\mathcal{B}} \circ f \circ \operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle^* \top \neq \bot \Leftrightarrow\\
&\left\langle \operatorname{id}^{\mathsf{FCD}}_{\mathcal{B}} \right\rangle \langle f \rangle \left\langle \operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}} \right\rangle^* \top \neq \bot \Leftrightarrow\\
&\mathcal{B} \sqcap \langle f \rangle (\mathcal{A} \sqcap \top) \neq \bot \Leftrightarrow\\
&\mathcal{B} \sqcap \langle f \rangle \mathcal{A} \neq \bot \Leftrightarrow\\
&\mathcal{A}\ [f]\ \mathcal{B}.
\end{aligned}$$

□

COROLLARY 887. Every filtrator of funcoids is star-separable.

PROOF. The set of funcoidal products of principal filters is a separation subset of the lattice of funcoids. □

THEOREM 888. Let A, B be sets. If $S \in \mathscr{P}(\mathscr{F}(A) \times \mathscr{F}(B))$ then

$$\prod_{(\mathcal{A},\mathcal{B}) \in S} (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) = \prod \operatorname{dom} S \times^{\mathsf{FCD}} \prod \operatorname{im} S.$$

PROOF. If $x \in \operatorname{atoms}^{\mathscr{F}(A)}$ then by theorem 875

$$\left\langle \prod_{(\mathcal{A},\mathcal{B}) \in S} (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \right\rangle x = \prod_{(\mathcal{A},\mathcal{B}) \in S} \langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle x.$$

If $x \not\asymp \prod \operatorname{dom} S$ then

$$\forall (\mathcal{A},\mathcal{B}) \in S : (x \sqcap \mathcal{A} \neq \bot \wedge \langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle x = \mathcal{B});$$

$$\left\{ \frac{\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle x}{(\mathcal{A},\mathcal{B}) \in S} \right\} = \operatorname{im} S;$$

if $x \asymp \prod \operatorname{dom} S$ then

$$\exists (\mathcal{A},\mathcal{B}) \in S : (x \sqcap \mathcal{A} = \bot \wedge \langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle x = \bot);$$

$$\left\{ \frac{\langle \mathcal{A} \times^{\mathsf{FCD}} \mathcal{B} \rangle x}{(\mathcal{A},\mathcal{B}) \in S} \right\} \ni \bot.$$

So

$$\left\langle \prod_{(\mathcal{A},\mathcal{B}) \in S} (\mathcal{A} \times^{\mathsf{FCD}} \mathcal{B}) \right\rangle x = \begin{cases} \prod \operatorname{im} S & \text{if } x \not\asymp \prod \operatorname{dom} S \\ \bot^{\mathscr{F}(B)} & \text{if } x \asymp \prod \operatorname{dom} S. \end{cases}$$

From this the statement of the theorem follows. □

COROLLARY 889. For every $\mathcal{A}_0, \mathcal{A}_1 \in \mathscr{F}(A)$, $\mathcal{B}_0, \mathcal{B}_1 \in \mathscr{F}(B)$ (for every sets A, B)

$$(\mathcal{A}_0 \times^{\mathsf{FCD}} \mathcal{B}_0) \sqcap (\mathcal{A}_1 \times^{\mathsf{FCD}} \mathcal{B}_1) = (\mathcal{A}_0 \sqcap \mathcal{A}_1) \times^{\mathsf{FCD}} (\mathcal{B}_0 \sqcap \mathcal{B}_1).$$