

1°. Consider the function $\alpha' \in \mathcal{F}(B)^{\mathcal{T}A}$ defined by the formula (for every $X \in \mathcal{T}A$)

$$\alpha' X = \bigsqcup \langle \alpha \rangle^* \text{atoms} \uparrow X.$$

Obviously $\alpha' \perp_{\mathcal{T}A} = \perp_{\mathcal{F}(B)}$. For every $I, J \in \mathcal{T}A$

$$\begin{aligned} \alpha'(I \sqcup J) &= \\ \bigsqcup \langle \alpha \rangle^* \text{atoms} \uparrow (I \sqcup J) &= \\ \bigsqcup \langle \alpha \rangle^* (\text{atoms} \uparrow \cup \text{atoms} \uparrow J) &= \\ \bigsqcup (\langle \alpha \rangle^* \text{atoms} \uparrow I \cup \langle \alpha \rangle^* \text{atoms} \uparrow J) &= \\ \bigsqcup \langle \alpha \rangle^* \text{atoms} \uparrow I \sqcup \bigsqcup \langle \alpha \rangle^* \text{atoms} \uparrow J &= \\ \alpha' I \sqcup \alpha' J. & \end{aligned}$$

Let continue α' till a funcoid f (by the theorem 828): $\langle f \rangle \mathcal{X} = \prod \langle \alpha' \rangle^* \text{up } \mathcal{X}$.
Let's prove the reverse of (10):

$$\begin{aligned} \prod \langle \bigsqcup \langle \alpha \rangle^* \circ \text{atoms} \circ \uparrow \rangle^* \text{up } a &= \\ \prod \langle \bigsqcup \langle \alpha \rangle^* \rangle^* \langle \text{atoms} \rangle^* \langle \uparrow \rangle^* \text{up } a &\sqsubseteq \\ \prod \langle \bigsqcup \langle \alpha \rangle^* \rangle^* \{ \{ a \} \} &= \\ \prod \{ (\bigsqcup \langle \alpha \rangle^*) \{ a \} \} &= \\ \prod \{ \bigsqcup \langle \alpha \rangle^* \{ a \} \} &= \\ \prod \{ \bigsqcup \{ \alpha a \} \} &= \\ \prod \{ \alpha a \} &= \\ \alpha a. & \end{aligned}$$

Finally,

$$\alpha a = \prod \langle \bigsqcup \langle \alpha \rangle^* \circ \text{atoms} \circ \uparrow \rangle^* \text{up } a = \prod \langle \alpha' \rangle^* \text{up } a = \langle f \rangle a,$$

so $\langle f \rangle$ is a continuation of α .

2°. Consider the relation $\delta' \in \mathcal{P}(\mathcal{T}A \times \mathcal{T}B)$ defined by the formula (for every $X \in \mathcal{T}A, Y \in \mathcal{T}B$)

$$X \delta' Y \Leftrightarrow \exists x \in \text{atoms} \uparrow X, y \in \text{atoms} \uparrow Y : x \delta y.$$

Obviously $\neg(X \delta' \perp_{\mathcal{T}B})$ and $\neg(\perp_{\mathcal{F}(A)} \delta' Y)$.

For suitable I and J we have:

$$\begin{aligned} I \sqcup J \delta' Y &\Leftrightarrow \\ \exists x \in \text{atoms} \uparrow (I \sqcup J), y \in \text{atoms} \uparrow Y : x \delta y &\Leftrightarrow \\ \exists x \in \text{atoms} \uparrow I \cup \text{atoms} \uparrow J, y \in \text{atoms} \uparrow Y : x \delta y &\Leftrightarrow \\ \exists x \in \text{atoms} \uparrow I, y \in \text{atoms} \uparrow Y : x \delta y \vee \exists x \in \text{atoms} \uparrow J, y \in \text{atoms} \uparrow Y : x \delta y &\Leftrightarrow \\ I \delta' Y \vee J \delta' Y; & \end{aligned}$$

similarly $X \delta' I \sqcup J \Leftrightarrow X \delta' I \vee X \delta' J$ for suitable I and J . Let's continue δ' till a funcoid f (by the theorem 828):

$$\mathcal{X} [f] \mathcal{Y} \Leftrightarrow \forall X \in \text{up } \mathcal{X}, Y \in \text{up } \mathcal{Y} : X \delta' Y.$$