PROOF. For every $\mathcal{Y} \in \mathscr{F}(\operatorname{Dst} f)$ we have

$$\begin{aligned} \mathcal{Y} \sqcap \langle f\rangle(\mathcal{X} \sqcap \operatorname{dom} f) \neq \bot \Leftrightarrow \\ \mathcal{X} \sqcap \operatorname{dom} f \sqcap \left\langle f^{-1}\right\rangle \mathcal{Y} \neq \bot \Leftrightarrow \\ \mathcal{X} \sqcap \operatorname{im} f^{-1} \sqcap \left\langle f^{-1}\right\rangle \mathcal{Y} \neq \bot \Leftrightarrow \\ \mathcal{X} \sqcap \left\langle f^{-1}\right\rangle \mathcal{Y} \neq \bot \Leftrightarrow \\ \mathcal{Y} \sqcap \langle f\rangle \mathcal{X} \neq \bot. \end{aligned}$$

Thus $\langle f\rangle(\mathcal{X} \sqcap \operatorname{dom} f) = \langle f\rangle \mathcal{X}$ because the lattice of filters is separable. □

PROPOSITION 865. $\langle f\rangle \mathcal{X} = \operatorname{im}(f|_{\mathcal{X}})$ for every funcoid f, $\mathcal{X} \in \mathscr{F}(\operatorname{Src} f)$.

PROOF.

$$\begin{aligned} \operatorname{im}(f|_{\mathcal{X}}) = \\ \left\langle f \circ \operatorname{id}^{\mathsf{FCD}}_{\mathcal{X}}\right\rangle \top = \\ \langle f\rangle \left\langle \operatorname{id}^{\mathsf{FCD}}_{\mathcal{X}}\right\rangle \top = \\ \langle f\rangle(\mathcal{X} \sqcap \top) = \\ \langle f\rangle \mathcal{X}. \end{aligned}$$

□

PROPOSITION 866. $\mathcal{X} \sqcap \operatorname{dom} f \neq \bot \Leftrightarrow \langle f\rangle \mathcal{X} \neq \bot$ for every funcoid f and $\mathcal{X} \in \mathscr{F}(\operatorname{Src} f)$.

PROOF.

$$\begin{aligned} \mathcal{X} \sqcap \operatorname{dom} f \neq \bot \Leftrightarrow \\ \mathcal{X} \sqcap \left\langle f^{-1}\right\rangle \top^{\mathscr{F}(\operatorname{Dst} f)} \neq \bot \Leftrightarrow \\ \top \sqcap \langle f\rangle \mathcal{X} \neq \bot \Leftrightarrow \\ \langle f\rangle \mathcal{X} \neq \bot. \end{aligned}$$

□

COROLLARY 867. $\operatorname{dom} f = \bigsqcup \left\{ \frac{a \in \operatorname{atoms}^{\mathscr{F}(\operatorname{Src} f)}}{\langle f\rangle a \neq \bot} \right\}$.

PROOF. This follows from the fact that $\mathscr{F}(\operatorname{Src} f)$ is an atomistic lattice. □

PROPOSITION 868. $\operatorname{dom}(f|_{\mathcal{A}}) = \mathcal{A} \sqcap \operatorname{dom} f$ for every funcoid f and $\mathcal{A} \in \mathscr{F}(\operatorname{Src} f)$.

PROOF.

$$\begin{aligned} \operatorname{dom}(f|_{\mathcal{A}}) = \\ \operatorname{im}(\operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}} \circ f^{-1}) = \\ \left\langle \operatorname{id}^{\mathsf{FCD}}_{\mathcal{A}}\right\rangle \left\langle f^{-1}\right\rangle \top = \\ \mathcal{A} \sqcap \left\langle f^{-1}\right\rangle \top = \\ \mathcal{A} \sqcap \operatorname{dom} f. \end{aligned}$$

□

THEOREM 869. $\operatorname{im} f = \bigsqcap^{\mathscr{F}} \langle \operatorname{im}\rangle^* \operatorname{up} f$ and $\operatorname{dom} f = \bigsqcap^{\mathscr{F}} \langle \operatorname{dom}\rangle^* \operatorname{up} f$ for every funcoid f.