CHAPTER 6

Common knowledge, part 2 (topology)

In this chapter I describe basics of the theory known as *general topology*. Starting with the next chapter after this one I will describe generalizations of customary objects of general topology described in this chapter.

The reason why I've written this chapter is to show to the reader kinds of objects which I generalize below in this book. For example, funcoids and a generalization of proximity spaces, and funcoids are a generalization of pretopologies. To understand the intuitive meaning of funcoids one needs first know what are proximities and what are pretopologies.

Having said that, customary topology is *not* used in my definitions and proofs below. It is just to feed your intuition.

6.1. Metric spaces

The theory of topological spaces started immediately with the definition would be completely non-intuitive for the reader. It is the reason why I first describe metric spaces and show that metric spaces give rise for a topology (see below). Topological spaces are understandable as a generalization of topologies induced by metric spaces.

Metric spaces is a formal way to express the notion of *distance*. For example, there are distance $|x-y|$ between real numbers x and y, distance between points of a plane, etc.

DEFINITION 761. A *metric space* is a set U together with a function $d : U\times U \rightarrow \mathbb{R}$ (*distance* or *metric*) such that for every $x, y, z \in U$:

1°. $d(x, y) \geq 0$;
2°. $d(x, y) = 0 \Leftrightarrow x = y$;
3°. $d(x, y) = d(y, x)$ (*symmetry*);
4°. $d(x, z) \leq d(x, y) + d(y, z)$ (*triangle inequality*).

EXERCISE 762. Show that the Euclid space $\mathbb{R}^n$ (with the standard distance) is a metric space for every $n \in \mathbb{N}$.

DEFINITION 763. *Open ball* of *radius* $r > 0$ centered at point $a \in U$ is the set

$$B_r(a) = \left\{ \frac{x \in U}{d(a, x) < r} \right\}.$$

DEFINITION 764. *Closed ball* of *radius* $r > 0$ centered at point $a \in U$ is the set

$$B_r[a] = \left\{ \frac{x \in U}{d(a, x) \leq r} \right\}.$$

One example of use of metric spaces: *Limit* of a sequence x in a metric space can be defined as a point y in this space such that

$$\forall \epsilon > 0 \exists N \in \mathbb{N} \forall n > N : d(x_n, y) < \epsilon.$$