PROPOSITION 653. Let $(\mathfrak{A}_i, \mathfrak{Z}_i)$ be filtrators and each $\mathfrak{Z}_i$ be a complete lattice with $\operatorname{up} x \neq \emptyset$ for every $x \in \mathfrak{A}_i$ (for every $i \in n$). For $a \in \prod \mathfrak{A}$:

1°. $\operatorname{Cor} a = \lambda i \in \operatorname{dom} a : \operatorname{Cor} a_i$;
2°. $\operatorname{Cor}' a = \lambda i \in \operatorname{dom} a : \operatorname{Cor}' a_i$.

PROOF. We will prove only the first, because the second is dual.

$$\begin{aligned}
&\operatorname{Cor} a = \\
&\bigsqcap^{\prod \mathfrak{Z}} \operatorname{up} a = \\
\lambda i \in \operatorname{dom} a : &\bigsqcap^{\mathfrak{Z}_i} \left\{ \frac{x_i}{x \in \operatorname{up} a} \right\} = (\operatorname{up} x \neq \emptyset \text{ taken into account}) \\
\lambda i \in \operatorname{dom} a : &\bigsqcap^{\mathfrak{Z}_i} \left\{ \frac{x}{x \in \operatorname{up} a_i} \right\} = \\
\lambda i \in \operatorname{dom} a : &\bigsqcap^{\mathfrak{Z}_i} \operatorname{up} a_i = \\
\lambda i \in \operatorname{dom} a : &\operatorname{Cor} a_i.
\end{aligned}$$

□

PROPOSITION 654. If each $(\mathfrak{A}_i, \mathfrak{Z}_i)$ is a filtrator with (co)separable core and each $\mathfrak{A}_i$ has a least (greatest) element, then $(\prod \mathfrak{A}, \prod \mathfrak{Z})$ is a filtrator with (co)separable core.

PROOF. We will prove only for separable core, as co-separable core is dual.

$$\begin{aligned}
&x \asymp^{\prod \mathfrak{A}} y \Leftrightarrow \\
&\text{(used the fact that } \mathfrak{A}_i \text{ has a least element)} \\
&\forall i \in \operatorname{dom} \mathfrak{A} : x_i \asymp^{\mathfrak{A}_i} y_i \Rightarrow \\
&\forall i \in \operatorname{dom} \mathfrak{A} \exists X \in \operatorname{up} x_i : X \asymp^{\mathfrak{A}_i} y_i \Leftrightarrow \\
&\exists X \in \operatorname{up} x \forall i \in \operatorname{dom} \mathfrak{A} : X_i \asymp^{\mathfrak{A}_i} y_i \Leftrightarrow \\
&\exists X \in \operatorname{up} x : X \asymp^{\prod \mathfrak{A}} y
\end{aligned}$$

for every $x, y \in \prod \mathfrak{A}$. □

OBVIOUS 655.

1°. If each $(\mathfrak{A}_i, \mathfrak{Z}_i)$ is a down-aligned filtrator, then $(\prod \mathfrak{A}, \prod \mathfrak{Z})$ is a down-aligned filtrator.
2°. If each $(\mathfrak{A}_i, \mathfrak{Z}_i)$ is an up-aligned filtrator, then $(\prod \mathfrak{A}, \prod \mathfrak{Z})$ is an up-aligned filtrator.

OBVIOUS 656.

1°. If each $(\mathfrak{A}_i, \mathfrak{Z}_i)$ is a weakly down-aligned filtrator, then $(\prod \mathfrak{A}, \prod \mathfrak{Z})$ is a weakly down-aligned filtrator.
2°. If each $(\mathfrak{A}_i, \mathfrak{Z}_i)$ is a weakly up-aligned filtrator, then $(\prod \mathfrak{A}, \prod \mathfrak{Z})$ is a weakly up-aligned filtrator.

PROPOSITION 657. If every b_i is substractive from a_i where a and b are n-indexed families of elements of distributive lattices with least elements (where n is an index set), then $a \setminus b = \lambda i \in n : a_i \setminus b_i$.