3°. $(\mathfrak{A},\mathfrak{Z})$ is a filtered down-aligned and up-aligned complete lattice filtrator with binarily meet-closed, separable and co-separable core which is a complete boolean lattice.
4°. $(a \sqcap^{\mathfrak{A}} b)^* = (a \sqcap^{\mathfrak{A}} b)^+ = a^* \sqcup^{\mathfrak{A}} b^* = a^+ \sqcup^{\mathfrak{A}} b^+$ for every $a, b \in \mathfrak{A}$.

PROOF.

1°⇒2°. Obvious.

2°⇒3°. The filtrator $(\mathfrak{A},\mathfrak{Z})$ is filtered by the theorem 531. $\mathfrak{A}$ is a complete lattice by corollary 515. $(\mathfrak{A},\mathfrak{Z})$ is with co-separable core by theorem 587. $(\mathfrak{A},\mathfrak{Z})$ is binarily meet-closed by proposition 533, with separable core by theorem 534.

3°⇒4°. Theorem 592 apply. Also theorem 598 apply because every filtered filtrator is join-closed. So

$$(a\sqcap^{\mathfrak{A}}b)^* = (a\sqcap^{\mathfrak{A}}b)^+ = \overline{\operatorname{Cor}(a \sqcap^{\mathfrak{A}} b)} = \overline{\operatorname{Cor} a \sqcap^{\mathfrak{Z}} \operatorname{Cor} b} = \overline{\operatorname{Cor} a}\sqcup^{\mathfrak{A}}\overline{\operatorname{Cor} b} = a^+\sqcup^{\mathfrak{A}}b^+ = a^*\sqcup^{\mathfrak{A}}b^*.$$

□

THEOREM 615. The following is an implications tuple:

1°. $(\mathfrak{A},\mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A},\mathfrak{Z})$ is a filtered starrish down-aligned and up-aligned complete lattice filtrator with binarily meet-closed, separable and co-separable core which is a complete atomistic boolean lattice.
3°. $(a \sqcup^{\mathfrak{A}} b)^* = (a \sqcup^{\mathfrak{A}} b)^+ = a^* \sqcap^{\mathfrak{A}} b^* = a^+ \sqcap^{\mathfrak{A}} b^+$ for every $a, b \in \mathfrak{A}$.

PROOF.

1°⇒2°. $(\mathfrak{A},\mathfrak{Z})$ is a filtered (theorem 531), distributive (corollary 528) complete lattice filtrator (corollary 515), with binarily meet-closed core (corollary 533), with separable core (theorem 534), with co-separable core (theorem 587).

2°⇒3°. $(a\sqcup^{\mathfrak{A}}b)^+ = (a\sqcup^{\mathfrak{A}}b)^* = \overline{\operatorname{Cor}'(a \sqcup^{\mathfrak{A}} b)} = \overline{\operatorname{Cor}' a \sqcup^{\mathfrak{Z}} \operatorname{Cor}' b} = \overline{\operatorname{Cor}' a}\sqcap^{\mathfrak{Z}}\overline{\operatorname{Cor}' b} = a^* \sqcap^{\mathfrak{Z}} b^* = a^* \sqcap^{\mathfrak{A}} b^* = a^+ \sqcap^{\mathfrak{A}} b^+$ (used theorems 591, 600, 592).

□

THEOREM 616. The following is an implications tuple:

1°. $(\mathfrak{A},\mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A},\mathfrak{Z})$ is a primary filtrator over a complete boolean lattice.
3°. $(\mathfrak{A},\mathfrak{Z})$ is a filtered complete lattice filtrator with down-aligned, binarily meet-closed, separable core which is a complete boolean lattice.
4°. $(a \sqcap^{\mathfrak{A}} b)^* = a^* \sqcup^{\mathfrak{A}} b^*$ for every $a, b \in \mathfrak{A}$.

PROOF.

1°⇒2°. Obvious.

2°⇒3°. It is filtered by theorem 531. It is complete lattice filtrator by 515. It is with binarily meet-closed core (corollary 533), with separable core (theorem 534).

3°⇒4°. It is join closed because it is filtered. $(a \sqcap^{\mathfrak{A}} b)^* = \overline{\operatorname{Cor}'(a \sqcap^{\mathfrak{A}} b)} = \overline{\operatorname{Cor}' a \sqcap^{\mathfrak{Z}} \operatorname{Cor}' b} = \overline{\operatorname{Cor}' a} \sqcup^{\mathfrak{Z}} \overline{\operatorname{Cor}' b} = a^* \sqcup^{\mathfrak{Z}} b^* = a^* \sqcup^{\mathfrak{A}} b^*$ (theorems 598, 591).

□

THEOREM 617. The following is an implications tuple:

1°. $(\mathfrak{A},\mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A},\mathfrak{Z})$ is a filtered starrish down-aligned complete lattice filtrator with binarily meet-closed, separable core which is a complete atomistic boolean lattice.