3°. $(\mathfrak{A}, \mathfrak{Z})$ is a filtrator with correct intersection, with binarily meet-closed and separable core.
4°. $B \asymp^{\mathfrak{A}} \mathcal{A} \Leftrightarrow \overline{B} \sqsupseteq \mathcal{A}$ for every $B \in \mathfrak{Z}$, $\mathcal{A} \in \mathfrak{A}$.

PROOF.
1°⇒2°. Obvious.
2°⇒3°. Using proposition 546, corollary 533, theorem 534.
3°⇒4°. By the lemma 548.

□

THEOREM 602. The following is an implications tuple:
1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a complete boolean lattice.
3°. $(\mathfrak{A}, \mathfrak{Z})$ is a filtrator over a boolean lattice with correct joining and co-separable core.
4°. $B \equiv^{\mathfrak{A}} \mathcal{A} \Leftrightarrow \overline{B} \sqsubseteq \mathcal{A}$ for every $B \in \mathfrak{Z}$, $\mathcal{A} \in \mathfrak{A}$.

PROOF.
1°⇒2°. Obvious.
2°⇒3°. Using obvious 547, theorem 587.
3°⇒4°. By the lemma 548.

□

5.25. Filtrators over Boolean Lattices

PROPOSITION 603. The following is an implications tuple:
1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a boolean lattice.
3°. $(\mathfrak{A}, \mathfrak{Z})$ is a down-aligned and up-aligned binarily meet-closed and binarily join-closed distributive lattice filtrator and $\mathfrak{Z}$ is a boolean lattice.
4°. $a \setminus^{\mathfrak{A}} B = a \sqcap^{\mathfrak{A}} \overline{B}$ for every $a \in \mathfrak{A}$, $B \in \mathfrak{Z}$.

PROOF.
1°⇒2°. Obvious.
2°⇒3°. $\mathfrak{A}$ is a distributive lattice by corollary 528. Our filtrator is binarily meet-closed by the corollary 533 and with join-closed core by the theorem 531. It is also up and down aligned.
3°⇒4°.

$$(a \sqcap^{\mathfrak{A}} \overline{B}) \sqcup^{\mathfrak{A}} B = (a \sqcup^{\mathfrak{A}} B) \sqcap^{\mathfrak{A}} (\overline{B} \sqcup^{\mathfrak{A}} B) = (a \sqcup^{\mathfrak{A}} B) \sqcap^{\mathfrak{A}} (\overline{B} \sqcup^{\mathfrak{Z}} B) = (a \sqcup^{\mathfrak{A}} B) \sqcap^{\mathfrak{A}} \top = a \sqcup^{\mathfrak{A}} B.$$

$$(a \sqcap^{\mathfrak{A}} \overline{B}) \sqcap^{\mathfrak{A}} B = a \sqcap^{\mathfrak{A}} (\overline{B} \sqcap^{\mathfrak{A}} B) = a \sqcap^{\mathfrak{A}} (\overline{B} \sqcap^{\mathfrak{Z}} B) = a \sqcap^{\mathfrak{A}} \bot = \bot.$$

So $a \sqcap^{\mathfrak{A}} \overline{B}$ is the difference of a and B.

□

PROPOSITION 604. For a primary filtrator over a complete boolean lattice both edge part and dual edge part are always defined.

PROOF. Core part and dual core part are defined because the core is a complete lattice. Using the theorem 603. □

THEOREM 605. The following is an implications tuple:
1°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a boolean lattice.
2°. $(\mathfrak{A}, \mathfrak{Z})$ is a complete co-brouwerian atomistic down-aligned lattice filtrator with binarily meet-closed and separable boolean core.