2°. $(\mathfrak{A}, \mathfrak{Z})$ is a complete lattice filtrator with join-closed separable core which is a complete lattice.

3°. $a^* \in \mathfrak{Z}$ for every $a \in \mathfrak{A}$.

Proof.

1°⇒2°. $\mathfrak{A}$ is a complete lattice by corollary 515. $(\mathfrak{A}, \mathfrak{Z})$ is a filtrator with join-closed core by theorem 531. $(\mathfrak{A}, \mathfrak{Z})$ is a filtrator with separable core by theorem 534.

2°⇒3°. $\left\{ \frac{c \in \mathfrak{A}}{c \sqcap^{\mathfrak{A}} a = \bot^{\mathfrak{A}}} \right\} \supseteq \left\{ \frac{A \in \mathfrak{Z}}{A \sqcap^{\mathfrak{A}} a = \bot^{\mathfrak{A}}} \right\}$; consequently $a^* \sqsupseteq \bigsqcup^{\mathfrak{A}} \left\{ \frac{A \in \mathfrak{Z}}{A \sqcap^{\mathfrak{A}} a = \bot^{\mathfrak{A}}} \right\}$.

But if $c \in \left\{ \frac{c \in \mathfrak{A}}{c \sqcap^{\mathfrak{A}} a = \bot^{\mathfrak{A}}} \right\}$ then there exists $A \in \mathfrak{Z}$ such that $A \sqsupseteq c$ and $A \sqcap^{\mathfrak{A}} a = \bot^{\mathfrak{A}}$ that is $A \in \left\{ \frac{A \in \mathfrak{Z}}{A \sqcap^{\mathfrak{A}} a = \bot^{\mathfrak{A}}} \right\}$. Consequently $a^* \sqsubseteq \bigsqcup^{\mathfrak{A}} \left\{ \frac{A \in \mathfrak{Z}}{A \sqcap^{\mathfrak{A}} a = \bot^{\mathfrak{A}}} \right\}$.

We have $a^* = \bigsqcup^{\mathfrak{A}} \left\{ \frac{A \in \mathfrak{Z}}{A \sqcap^{\mathfrak{A}} a = \bot^{\mathfrak{A}}} \right\} = \bigsqcup^{\mathfrak{Z}} \left\{ \frac{A \in \mathfrak{Z}}{A \sqcap^{\mathfrak{A}} a = \bot^{\mathfrak{A}}} \right\} \in \mathfrak{Z}$.

□

Theorem 594. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.

2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over a complete boolean lattice.

3°. $(\mathfrak{A}, \mathfrak{Z})$ is an up-aligned filtered complete lattice filtrator with co-separable core which is a complete boolean lattice.

4°. a^+ is dual pseudocomplement of a, that is

$$a^+ = \min \left\{ \frac{c \in \mathfrak{A}}{c \sqcup^{\mathfrak{A}} a = \top^{\mathfrak{A}}} \right\}$$

for every $a \in \mathfrak{A}$.

Proof.

1°⇒2°. Obvious.

2°⇒3°. $(\mathfrak{A}, \mathfrak{Z})$ is filtered by the theorem 531. It is with co-separable core by theorem 587. $\mathfrak{A}$ is a complete lattice by corollary 515.

3°⇒4°. Our filtrator is with join-closed core (theorem 531). It's enough to prove that $a^+ \sqcup^{\mathfrak{A}} a = \top^{\mathfrak{A}}$. But $a^+ \sqcup^{\mathfrak{A}} a = \overline{\operatorname{Cor} a} \sqcup^{\mathfrak{A}} a \sqsupseteq \overline{\operatorname{Cor} a} \sqcup^{\mathfrak{A}} \operatorname{Cor} a = \overline{\operatorname{Cor} a} \sqcup^{\mathfrak{Z}} \operatorname{Cor} a = \top^{\mathfrak{A}}$ (used the theorem 539 and the fact that our filtrator is filtered).

□

Definition 595. The *edge part* of an element $a \in \mathfrak{A}$ is $\operatorname{Edg} a = a \setminus \operatorname{Cor} a$, the *dual edge part* is $\operatorname{Edg}' a = a \setminus \operatorname{Cor}' a$.

Knowing core part and edge part or dual core part and dual edge part of an element of a filtrator, the filter can be restored by the formulas:

$$a = \operatorname{Cor} a \sqcup^{\mathfrak{A}} \operatorname{Edg} a \quad \text{and} \quad a = \operatorname{Cor}' a \sqcup^{\mathfrak{A}} \operatorname{Edg}' a.$$

5.22. Core Part and Atomic Elements

Proposition 596. The following is an implications tuple:

1°. $(\mathfrak{A}, \mathfrak{Z})$ is a powerset filtrator.

2°. $(\mathfrak{A}, \mathfrak{Z})$ is a primary filtrator over an atomistic lattice.

3°. $(\mathfrak{A}, \mathfrak{Z})$ is a filtrator with join-closed core and $\mathfrak{Z}$ be an atomistic lattice.

4°. $\operatorname{Cor}' a = \bigsqcup^{\mathfrak{Z}} \left\{ \frac{x}{x \text{ is an atom of } \mathfrak{Z}, x \sqsubseteq a} \right\}$ for every $a \in \mathfrak{A}$ such that $\operatorname{Cor}' a$ exists.

Proof.

1°⇒2°. Obvious.

2°⇒3°. $(\mathfrak{A}, \mathfrak{Z})$ is with join-closed core by corollary 531.